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**Summary:**  
This deliverable describes the outcome of the Aeolus project task 3.3. It is divided in two parts: *Wind turbine model* and *Reconfigurable control paradigm*.  
The first part of the deliverable describes the derivation of the wind turbine model in a form that is appropriate for the chosen control approach. Two wind turbine models are studied: the Vestas’ V80 wind turbine model and the wind turbine model produced by National Renewable Energy Laboratory (NREL).  
The second part of the deliverable addresses the reconfigurable control paradigm. It describes the set-up of the reconfigurable wind farm control paradigm, its theoretical background and the obtained simulation results.
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Distributed Control of Large-Scale Offshore Wind Farms  
(AEOLUS)

Deliverable 3.3:  
Reconfigurable control extension

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Abstract

This deliverable describes the outcome of the Aeolus project task 3.3. It is divided in two parts: *Wind turbine model* and *Reconfigurable control paradigm.*

The first part of the deliverable describes the derivation of the wind turbine model in a form that is appropriate for the chosen control approach. Two wind turbine models are studied: the Vestas’ V80 wind turbine model and the wind turbine model produced by National Renewable Energy Laboratory (NREL).

The second part of the deliverable addresses the reconfigurable control paradigm. It describes the set-up of the reconfigurable wind farm control paradigm, its theoretical background and the obtained simulation results.
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Part I

Wind turbine model
Chapter 1

Introduction

This part of the deliverable describes the analysis of the wind turbine models provided in Work Package 5 and the accommodation of the models to the form required for the reconfigurable control design and implementation. Two different models are analyzed, the model of the V80 wind turbine provided by Vestas (V90 wind turbine can be modeled analogously, only the parameters would be different) and the wind turbine model by the National Renewable Energy Laboratory (NREL). Both of these wind turbines are variable speed wind turbines with active pitch control and Doubly-Fed Induction Generator (DFIG) with controllable generator torque. Even though their mechanical and electrical parts are very similar, their control systems are, to a certain extent, different. Also, the large issue in this task was the fact that the control system of the V80 wind turbine is undisclosed, so it had to be identified in order to obtain a valid model.

The provided wind turbine models are nonlinear hybrid models. The hybrid nature of the model emerges from the control system that switches between different control paradigms. This is typical for the wind turbine control. Depending on the availability of required wind speed the wind turbine control system selects between the maximization of the produced power and tracking the desired power. Also, the constraints on the generator speed and torque impose different strategies for power maximization at different operating conditions. The provided models in their original form are inadequate for the design and implementation of a reconfigurable controller.

To obtain wind turbine models suitable for controller design, firstly the nonlinear static models of the wind turbines were derived. They are the baseline for the linearization of the system dynamics around the operating points. The dynamical model obtained in this way was further simplified by disregarding fast dynamics that is irrelevant on the wind farm control level.

Since the general static and dynamic models were constructed by analytic approach (instead of black box identification of the system in certain operating points), it is possible to produce as many linearized models of the system as will be required by the control design to obtain the best performance. This also provides large freedom for future work that could be based on piecewise-affine description of the system dynamics. Also, the model obtained
in this way offers a very good insight in the behavior of the wind turbines.

In Chapter 2 the subsystems that build up wind turbine models will be presented.

In Chapter 3 the wind turbine control system is analyzed. Most of the V80 control system has not been readily available in the model provided by Vestas. To obtain an overall dynamic model the functionality of the undisclosed models was identified through different identification experiments. The results are presented.

In Chapter 4 the static models of the wind turbines is derived. The hybrid nature of wind turbine models is discussed.

In Chapter 5 the simplified dynamic models of the wind turbines are derived. These models are obtained by neglecting the dynamics that is too fast for control on wind farm level. The obtained results are the state-space models of low complexity that are suitable for reconfigurable control design.

The Chapter 6 concludes the work described in Part I of the Deliverable.
Chapter 2

Wind turbine subsystems

The general wind turbine model is shown in Fig. 2.1. It consists of the following subsystems:

- Rotor aerodynamics,
- Electrical generator,
- Transmission system,
- Tower nodding,
- Pitch controller and pitch servo system,
- Generator power controller, and
- Wind turbine local controller.

In this chapter the overview of the mechanical and electrical subsystems of the wind turbine model is provided. These subsystems are the same in both V80 and NREL wind turbines. The wind turbine local controller for each of the wind turbines will be presented in the next chapter.
Figure 2.1: Wind turbine model
2.1. Rotor aerodynamics

The most essential part of wind turbine model is the model of aerodynamic conversion, which describes how the energy captured by the rotor converts to driving torque of the rotating machine. Details on this conversion can be found in various references, e.g. [1].

In the simulation model this conversion is described by a static nonlinear equation resulting in the following expression for the wind turbine power $P_a$:

$$P_a = \frac{\pi}{2} \rho R^2 v^3 C_P(\lambda, \beta),$$  \hspace{1cm} (2.1)

where:

- $v$ [m/s] is wind speed,
- $\omega_r$ [rad/s] is rotational speed of wind turbine’s rotor,
- $\beta$ [°] is collective pitch angle,
- $\rho$ [kg/m$^3$] is air density, and
- $R$ [m] is radius of wind turbine rotor.

The power coefficient $C_P(\frac{\omega_r R}{v}, \beta)$ is a wind turbine specific nonlinear function in tip speed ratio, which is defined as $\lambda = \frac{\omega_r R}{v}$, and pitch angle. Power coefficient curves for this wind turbine are shown in Fig. 2.2.

![Power coefficient curves](image)

Figure 2.2: Power coefficient of a wind turbine parameterized by pitch angle

The aerodynamical torque $T_r$ applied on the rotor shaft can be derived from aerodynamic power according to the relation:

$$T_r = \frac{P_a}{\omega_r},$$  \hspace{1cm} (2.2)
If a torque coefficient $C_Q(\lambda, \beta) = \frac{C_P(\lambda, \beta)}{\lambda}$ is introduced to express the aerodynamical torque, the following relation applies:

$$T_r = \frac{\pi R^3}{2} \rho v^2 C_Q(\lambda, \beta). \quad (2.3)$$

A by-product of energy conversion that takes place at wind turbine rotor is the thrust force. It is the force that acts on wind turbine rotor in direction perpendicular to rotor plane, [2]. This force produces tower bending moment and causes wind turbine vibrations in fore-aft direction. Thrust force $F_t$ is modeled by the following nonlinear static relation:

$$F_t = \frac{\pi R^2 \rho}{2} C_T(\lambda, \beta) v^2. \quad (2.4)$$

The coefficient $C_T(\lambda, \beta)$ is referred to as thrust coefficient and it is, similar to $C_P(\lambda, \beta)$, a wind turbine specific nonlinear function.

### 2.2. Electrical generator

In the modeled wind turbine the conversion of mechanical to electrical energy takes place in a Doubly-Fed Induction Generator (DFIG). This type of generator is connected to the grid at both stator and rotor. The rotor is connected to the grid through two IGBT Voltage-source converters connected back-to-back, see Fig. 2.3. By controlling the voltage amplitude, phase angle and frequency of the rotor power supply, this type of generator enables operation at variable rotor speed, as well as independent control of active and reactive power, [3].

![Wind turbine grid connection](image)

**Figure 2.3: Wind turbine grid connection**

In the literature (e.g. [3]), where lossless generator is considered, the expression for active electrical power delivered by DFIG is:

$$P = P_{\text{stat}} + P_{\text{rot}}, \quad (2.5)$$

where $P_{\text{stat}}$ is real power at stator terminal and $P_{\text{rot}}$ is real power at rotor terminal. From induction machine theory the airgap power equals, [3]:

$$P_{\text{airgap}} = T_g \cdot \omega_S, \quad (2.6)$$
where $\omega_S$ is the frequency of the stator AC supply and $T_g$ generator torque. 
By neglecting the stator and rotor losses one can assume:

$$P_{\text{stat}} \approx P_{\text{airgap}}$$  \hspace{1cm} (2.7)$$

Power equilibrium for the rotor than derives:

$$P_{\text{rot}} \approx T_g \cdot p \cdot \omega_g - P_{\text{stat}} \approx s \cdot P_{\text{stat}},$$  \hspace{1cm} (2.8)$$

where $p$ is the number of pole pairs in the generator and $s$ is the rotor slip, $s = \frac{p \omega_S - \Omega_S}{\omega_S}$. 

Asynchronous operation of this type of generator is possible because the power electronic converter connected to the rotor supplies rotor windings with currents at slip frequency, $(\omega_S - p \cdot \omega_S)$. Thereby, the currents in the rotor produce magnetic flux that rotates at frequency $(p \omega_S + \omega_S - p \omega_S)$, i.e. in synchronism with the stator flux, [3].

If electrical frequency of the rotor is less than frequency of stator supply than rotor circuit consumes electrical power to speed-up the rotor flux. Therefore, rotor power is negative. In the opposite situation rotor delivers surplus power to the grid through the rotor circuit and thus decreases rotor flux frequency and rotor power is positive.

In the provided V80 simulation model the generator is described by the following static model, in which the generator losses are included to certain extent:

$$P = P_{\text{stat}} + P_{\text{rot}} - P_{\text{loss}},$$  \hspace{1cm} (2.9)$$

$$T_g = K_{PT} \cdot P_{\text{stat}},$$  \hspace{1cm} (2.10)$$

$$P_{\text{rot}} = s \cdot P_{\text{stat}},$$  \hspace{1cm} (2.11)$$

Here, $K_{PT}$ is a constant that describes proportional relation between $T_g$ and $P_{\text{stat}}$, but does not exactly match the constant from theoretical lossless model. The constant $P_{\text{loss}}$ describes idling consumption of the generator.

In this project the focus is on active power control at turbine (mechanical) level. Therefore, the power controller in the simulation model is not described in details. The stator power, $P_{\text{stat}}$, is considered a control variable. Further analysis of power controller will be given in Sec. 2.6.

The NREL model contains an even simpler description of the electrical generator. In this model it is assumed that the power controller provides the electrical generator with torque reference. The generator dynamics is modeled by the first order system:

$$\frac{T_g(s)}{T_{g\text{ref}}(s)} = \frac{1}{T_{\text{gen}} s + 1},$$  \hspace{1cm} (2.12)$$

where:

$T_{\text{gen}} [s]$ is the generator time constant.

The output electrical power is modeled by the nonlinear equation:

$$P = \omega_g \cdot T_g.$$  \hspace{1cm} (2.13)$$
2.3. Transmission system

The transmission system of wind turbine is modeled as a high-order linear system. Several phenomena are modeled:

- speed reduction at gearbox,
- stiffness and damping of the low speed shaft,
- transition of torque to wind turbine structure at connections of generator stator and gear-box to nacelle (included in V80 model only), and
- drive train losses (included in V80 model only).

The gearbox is not modeled in details, only as a gain with multiplication ratio \( n_{gb} \).

The low speed shaft is modeled as damped harmonic oscillator.

The dynamical model of the drive train of the NREL wind turbine is given by the following transfer functions:

\[
\frac{\Omega_r(s)}{T_r(s)} = \frac{n_{gb}^2 J_g s^2 + Bs + K}{s(J_r n_{gb}^2 J_g s^2 + B(J_r + n_{gb}^2 J_g)s + K(J_r + n_{gb}^2 J_g))},
\]

\[
\frac{\Omega_g(s)}{T_r(s)} = \frac{n_{gb}(Bs + K)}{s(J_r n_{gb}^2 J_g s^2 + B(J_r + n_{gb}^2 J_g)s + K(J_r + n_{gb}^2 J_g))},
\]

\[
\frac{\Omega_g(s)}{T_g(s)} = \frac{n_{gb}^2 J_g s^2 + Bs + K}{s(J_r n_{gb}^2 J_g s^2 + B(J_r + n_{gb}^2 J_g)s + K(J_r + n_{gb}^2 J_g))},
\]

\[
\frac{M_{shaft}(s)}{T_r(s)} = \frac{n_{gb}^2 J_g (Bs + K)}{s(J_r n_{gb}^2 J_g s^2 + B(J_r + n_{gb}^2 J_g)s + K(J_r + n_{gb}^2 J_g))},
\]

\[
\frac{M_{shaft}(s)}{T_g(s)} = \frac{n_{gb} J_r (Bs + K)}{s(J_r n_{gb}^2 J_g s^2 + B(J_r + n_{gb}^2 J_g)s + K(J_r + n_{gb}^2 J_g))},
\]

where:

\( J_r \ [kg \ m^2] \) is the rotor inertia,

\( J_g \ [kg \ m^2] \) is the generator inertia,

\( B \ [Nms/rad] \) is the main shaft viscous friction,

\( K \ [Nm/rad] \) is the main shaft spring constant, and

\( M_{shaft} \ [Nm] \) is the main shaft torque.
The V80 dynamical model is more complicated because it contains the model of the tower side-side motion and drive train losses.

Due to the connection of the generator stator and gearbox housing to the nacelle, a part of the generator torque is transferred to the tower and it excites tower side-side motion, [4]. This connection is presented in the diagram in Fig. 2.4. This mechanical system is described by a high order dynamical model. In the provided model certain simplifications were introduced, see [4], and the third order linear model that describes the system was derived, see [2].

![Diagram](image_url)

Figure 2.4: Drive train rotation and tower side-side motion modeling

The drive train losses are modeled as speed dependant and torque dependant losses, according to the following equation (see Fig. 2.5):

\[ T'_{g} = C_{\omega g} \cdot \omega_{g} + C_{T_{g}} \cdot T_{g}, \quad (2.20) \]

where coefficient \( C_{\omega g}(>0) \) describes speed dependant losses and \( C_{T_{g}}(>1) \) describes torque dependant losses.

The overall drive train model of the V80 wind turbine is a linear system of high order. The approximation of lower order will be derived in Ch. 5 and the comparison to the original model will be made based on its frequency characteristic (see Fig. 5.6).

The drive-train experiences large oscillations at drive-train eigenfrequency \( f_{DT} = 1.684 \, \text{Hz} \). These oscillations should be sufficiently damped by wind turbine local control system. The extensions to the wind turbine control system that assure the damping around this frequency will be described in Sec. 3.
2.4. Tower nodding

The thrust force, (2.4), acts on a wind turbine rotor in the direction perpendicular to the rotor plane. Due to elasticity of the wind turbine structure, this force produces wind turbine structure fore-aft oscillations.

In the provided models one mode of tower vibration is described. The following model for wind turbine fore-aft motion is derived:

\[ F_T(\lambda, \beta) = M_t \cdot \ddot{x} + D_t \cdot \dot{x} + C_t \cdot x, \quad (2.21) \]

where:

- \( x \) [m] is the nacelle displacement in wind direction,
- \( F_T \) [N] is the thrust force, (2.4),
- \( M_t \) [kg] is the modal mass of the tower,
- \( D_t \) [Ns/m] is the modal damping of the tower, and
- \( C_t \) [N/m] is the modal stiffness of the tower.

The fore-aft tower motion causes structure fatigue. Also, it influences the dynamic response of wind turbine states, because it enters the wind turbine model through notion of effective wind speed:

\[ v_{\text{eff}} = v - \dot{x}. \quad (2.22) \]

Oscillations in wind speed lead to oscillations in wind turbine states. Therefore, one of the primary tasks for the local wind turbine control system is to ensure damping to wind tower oscillations.
2.5. Pitch servo drive

Modern multi-megawatt wind turbines mostly achieve speed and power control by blade pitching. Such turbines need to be equipped with controlled servo drives that are capable to position each of the blades to the given setpoint provided by the wind turbine local controller. Wind turbine in scope of this work uses hydraulic pitch drive. The dynamics of hydraulic pitch drive is relatively slow process, especially when compared to the generator power control done by frequency converter. Therefore it is described in the model in more details. The structure of the subsystem is shown in Fig. 2.6. The subsystem includes model of the pitch mechanism nonlinearities and hydraulic dead time. Pitch angle is regulated in closed loop by a proportional controller. Saturation of the control voltage is also implemented as well as the effects of the pitch angle digital measuring device.

![Pitch controller and pitch servo system](image)

Figure 2.6: Pitch controller and pitch servo system

2.6. Generator power controller

Generator power controller tracks the reference power that is provided to it by the wind turbine local controller.

The NREL wind turbine model contains a very simple description of both the generator and its controller. The model of the generator is given by (2.12) and (2.13). The power controller simply translates the power demand provided to it by the local controller to the torque reference which is provided to the generator (it is actually provided to the frequency converter control system). The relation between the power reference and the torque reference is simply:

\[ T_{\text{ref}} = \frac{P_{\text{ref}}}{\omega_g}. \]  

(2.23)

The generator power controller of the V80 model is more complicated. It is a closed loop controller which produces the reference value for the stator power \( P_{\text{stat}}^{\text{ref}} \) as a control signal. The power reference \( P_{\text{stat}}^{\text{ref}} \) is generated based on the reference value for the electrical power and the measurements of the electrical power and the generator speed. Frequency converter
control actually controls rotor voltage and currents, but for the purpose of modeling the generator real power control at turbine related time scale (that is significantly slower than the time scale of electrical transients), these control signals can be uniquely translated to stator power control signal. Generator power controller of the V80 wind turbine structure is depicted in Fig. 2.7.

Main control action of the power controller is the integration of power error signal. This action actually destabilizes wind turbine in occurrence of wind gusts. Namely, the wind gust speeds up wind turbine rotor and generator, which by (2.9)–(2.11) increases wind turbine electrical power. At the same time, due to negative power error signal, power controller decreases stator power. According to (2.10) this leads to decrease in generator torque, which again speeds up the generator.

Additional functionality of the power controller is active damping of the transmission system oscillations by influencing the generator torque. For this purpose a compensating signal is added to the control signal produced by integrating controller, as can be seen in Fig. 2.7. In the provided model this compensator is contained in an undisclosed subsystem $UMPDAMP$. The transfer function of the system was therefore identified by several input-output identification procedures. The identified model is:

$$G_{UMPDAMP}(s) = \frac{U_c(s)}{\Omega_g(s)} = \frac{K_D s}{(T_1^D s + 1)(T_2^D s + 1)}.$$  \hspace{1cm} (2.24)

This feedback stabilizes the overall control system by increasing stator power and generator torque when generator speeds-up. Also, this feedback is designed in such way that it has highest gain at drive train eigenfrequency, see Fig. 2.8. Therefore, drive train oscillations are actively damped by generator control.

With this intervention the generator control has "taken-over" the speed control around drive train frequencies. To complement this approach, a Transmission filter subsystem is added to the generator speed feedback used within wind turbine speed controller. Identification experiments showed that this subsystem contains a notch filter with central frequency
adaptable to power reference. This adaptation was not analyzed further because it does not seem to have a large effect on wind turbine dynamic behavior. The Butterworth bandstop filter was designed that approximates the response of the transmission filter well. The cut-off frequencies were set to 1 Hz and 10 Hz. The best agreement with the original model was obtained for filter of fourth order. The magnitude response of the filter is given in Fig. 2.9.
Chapter 3

Wind turbine control system

The primary aim of a wind turbine control system is to enable the delivery of the required amount of power (which has to be less or equal to the generator nominal power). Since the wind turbine exerts the power from the wind stream, there is a different maximal amount of power available at each time instant. The maximal available power is a function of wind speed, wind turbine rotational speed and turbine's geometry (which includes blade pitch angle). Therefore, there exists a duality in wind turbine control objectives - the first is to deliver the demanded power, and the second is to maximize the power production if the demanded power is not available at given time. In this project two main inputs to the wind turbine model are considered - turbine demanded power and the wind speed. These two inputs uniquely determine the wind turbine operating regime.

Also, different constraints exist in wind turbine system - most importantly, the constraints on generator’s speed and torque. The wind turbine local control system has to make sure that those constraints are not violated. Furthermore, local wind turbine control system has a task to stabilize the behavior of the local subsystems - to damp tower and shaft oscillation.

A fact about wind turbine control that should be pointed out is the existence of two control variables - the pitch angle and the torque (or power) reference to the electrical generator. This allows a lot of freedom in control design and optimization of wind turbine behavior. The wind turbine control system is a hybrid nonlinear MIMO system. Based on the operating conditions, wind turbine control system switches between alternative configurations which satisfy certain objectives.

It is also important to notice that when the power maximization control mode is active, the demanded power (which is considered to be the only wind turbine control variable in the Aeolus project) has no influence on the wind turbine behavior. Therefore, a wind turbine in power maximization mode will only be a disturbance in a wind farm control system.

The control systems of V80 and NREL wind turbine will be described in the following sections.
3.1. V80 wind turbine control system

The V80 wind turbine control system differs two primary objectives:

1. to regulate the generator speed towards the speed reference that is obtained internally based on wind speed measurements, and
2. to regulate electrical power towards externally provided power reference.

The first objective remains the same in the entire wind turbine operating region. The speed control has an additional task to speed regulation and that is to provide satisfactory damping of tower oscillations. The second objective, however, can not always be accomplished. Due to the random nature of power resource availability it is not always possible to reach the desired references. Therefore, local control system has an alternative objective and that is to maximize power production.

These two objectives are not accomplished in the same way. In the provided wind turbine model there exist two entirely different control structures that correspond to the alternative tasks. At power regulation the pitch angle is used for speed regulation and the generator control is used to obtain required power demanded by an external power reference. On the other hand, at power maximization the power reference does not influence the regulation. Speed control is accomplished with the generator power and current controller as actuators and pitch angle is used to maximize the efficiency of the power extraction. A complex logic controls the switching between the different structures. The active structure is indicated by a logic variable $FULL$. $FULL = 1$ denotes the power control of the wind turbine while $FULL = 0$ denotes the maximization of the power output.

In this project the aim is to treat wind turbines as the wind farm’s power actuators. Therefore, the emphases in this work was put on identification of power control structure and conditions on power reference that ensure that power control structure was active.

Wind turbine controller is a digital controller with sampling time of 0.1 s. It consists of several subsystems, see Fig. 3.1:

- **Stationary control** - provides speed and power references to control loops,
- **Logic** - controls the switching between various control structures,
- **Full load controller** and **Partial load controller** - implement controllers for individual control structures,
- **OptiTip** - calculates the pitch angle at which maximal power for a given operating point can be extracted, and
- **Transmission filter** - filters out shaft oscillations from generator speed measurement, see Sec. 2.6.

All of these subsystems are undisclosed. Therefore, all the results described in the following sections are the results of the identification procedures performed on the controller. The aim
of these identifications is to deliver an overall wind turbine dynamic model. Since most of the
blocks contain nonlinearities such as saturation and gain scheduling, the chosen approach was
the so-called "grey-box modeling". In this approach certain assumptions based on the system
knowledge were made and the obtained measurements were fitted to the assumed model.
Figure 3.1: V80’s local wind turbine controller
### 3.1.1. Stationary control

The *Stationary control* subsystem determines the static behavior of the V80 wind turbine. It associates the measured wind speed with the rotational speed reference, $\Omega_{\text{ref}}$, [rpm]. The demanded power has no influence on the rotational speed reference. This is a dynamical subsystem. By performing multiple simulations the following model was identified:

\[
\omega'_{\text{ref}} = \begin{cases} 
\omega_{\text{min}}, & v_{\text{meas}} \leq v_{\omega_{\text{min}}} \\
 k \cdot v + l, & v_{\omega_{\text{min}}} < v_{\text{meas}} \leq v_{\omega_{\text{nom}}} \\
 \omega_{\text{nom}}, & v_{\text{meas}} > v_{\omega_{\text{nom}}}
\end{cases}
\]  

\[
\omega_{\text{ref}} = \frac{1}{T_{\text{scs}} + 1} \omega'_{\text{ref}},
\]

where:

$\omega_{\text{min}}$ - the minimal generator speed necessary to produce electrical power,

$\omega_{\text{nom}}$ - the nominal generator speed,

$v_{\omega_{\text{min}}}$ - the highest wind speed at which generator speed reference is $\omega_{\text{min}}$, and

$v_{\omega_{\text{nom}}}$ - the lowest wind speed at which generator speed reference is $\omega_{\text{nom}}$.

The static model (3.1) is shown on Fig. 3.2. Wind turbine cut-in and cut-out wind speeds are denoted in the figure.

![Figure 3.2: Static model of Stationary control](image-url)
The idea behind this static model is the maximization of wind turbine output. Namely, this characteristic ensures optimal tip-speed ratio, i.e. the tip-speed ratio at which wind turbine extracts maximal power, for wind speeds inside the interval \( [v_{\omega_{\text{min}}}, v_{\omega_{\text{max}}}] \). This interval is determined by limitations on rotor speed. So, this characteristic determines maximal power that can be extracted from a wind turbine. This characteristic remains the same in power control structure even though the objective that motivates the approach is abandoned.

For modeling of the stationary control it is also necessary to model the wind speed measuring device. In the provided model the measuring device significantly filters the wind speed entering the stationary control. The time constant of this filter is \( T_{\text{vm}} = 2 \text{ s} \). This time constant falls into the time scale interesting for reconfigurable control. The nonlinearities induced by measurement device resolution and sampling of the measurements are also modeled as well as the computational dead time. The model of wind speed measuring device is shown on Fig. 3.3.

Full load controller is a part of the local controller that controls the reference for collective pitch angle, see Fig. 3.1. Its functionality is dual:

- if the wind turbine is power controlled \( (FULL = 1) \) it implements the closed loop speed controller with pitch angle as a control variable, and
- if the wind turbine maximizes produced power \( (FULL = 0) \) it passes over the optimal pitch angle from OptiTip subsystem.

The functionality of the block for power maximization is trivial:

\[
\beta_{\text{ref}} = \beta_{\text{opt}}.
\] (3.3)

For power controlled wind turbine this block implements a closed loop regulation of generator speed. The equivalent structure for this subsystem and its parameters were identified through several identification experiments. The obtained structure is shown on Fig. 3.4.

A look-up table for \( K_P \) is given on Fig. 3.5.

Partial load controller is the local controller subsystem that controls the reference for generator power, see Fig. 3.1. Its functionality is, again, dual:
- if the wind turbine is power controlled \((FULL = 1)\) this subsystem simply passes over the power reference provided by the user, and

- if the wind turbine maximizes produced power \((FULL = 0)\) it contains the closed loop generator speed controller that uses electrical power reference as control signal.

The functionality of this subsystem for \(FULL = 1\) is trivial:

\[
    P_{\text{ref}} = P_{\text{dem}}. \tag{3.4}
\]

For power maximization this subsystem implements a closed loop speed controller. The details of this model were not identified because in this work the model of power controlled wind turbine is derived and this subsystem does not take part in that controller structure.

The control structures for power controlled wind turbine and maximization of power output are given on Fig. 3.6.
Figure 3.6: The structures of wind turbine control

(a) Power controlled wind turbine

(b) Wind turbine power optimization
3.1.4. OptiTip subsystem

The OptiTip subsystem is a dynamic subsystem that outputs optimal pitch angle, i.e. pitch angle at which wind turbine can extract maximal power, given the measured wind speed and generator speed. Since this subsystem is only a part of the power optimization control structure, it is not important to identify entirely its behavior. However, it takes part in the switching logic of the controller and therefore it is useful to know at least its static behavior. At steady state the tip speed ratio is uniquely determined by the wind speed and the functionality of the Stationary control subsystem. The optimal pitch angle is then the one which maximizes the power coefficient, see Fig. 2.2(b). Generator speed does not influence the static characteristic of the OptiTip subsystem.

3.1.5. Switching logic

The two alternative objectives of wind turbine control, power control of wind turbine and maximization of wind turbine power are motivated by two possible scenarios that can occur in wind turbine operation:

- the reference power is less than power available in wind, and

- the reference power is equal or larger than the available power.

The available power is the maximal power that wind turbine can extract from wind at a given wind speed reduced for wind turbine static losses. The Fig. 3.7 depicts the available power as a function of wind speed. The analytical model will be derived in the following chapter.

![Figure 3.7: Electrical power available at given wind speed](image-url)
The condition $P_{\text{ref}} \geq P_{\text{max}}$ defines the condition for power control structure, but only in the steady state. The dynamics of wind turbine are not taken into consideration. The switching paradigm of the control system takes into account the behavior of control loops to ensure stability and performance of the overall system.

The wind turbine controller implemented in Vestas V80 wind turbine uses several logic signals to initiate the switching of control structures. As previously described, selection of control structure is determined by the logical value of the flag $FULL$:

- $FULL = 1$ selects the power control structure, and
- $FULL = 0$ selects control structure for maximizing power extraction.

The three flags influence the logical value of $FULL$: $NOMPOW, \ RPM > \ REF$ and $THOPT$. The identification procedure yielded the following flag definitions:

$NOMPOW = \begin{cases} 1, & P_{\text{dem}} \geq P_{\text{ref}} \ & & & & & & & & \ & FULL = 0, \\ 0, & \text{otherwise,} \end{cases}$ \hfill (3.5)

$RPM > \ REF = \begin{cases} 1, & \omega_g > \omega_{\text{ref}}, \\ 0, & \text{otherwise,} \end{cases}$ \hfill (3.6)

$THOPT = \begin{cases} 1, & \beta_{\text{ref}} - \beta_{\text{opt}} < 0.5, \\ 0, & \text{otherwise.} \end{cases}$ \hfill (3.7)

The flag $NOMPOW$ signals that the power reference obtained in closed loop speed control in power optimization structure is larger than the external reference power. The flag $RPM > \ REF$ indicates that generator speed is larger than its reference. The flag $THOPT$ indicates that pitch angle is close to optimal pitch angle.

In Tab. 3.1 the identified functionality of the $Logic$ subsystem is provided.

<table>
<thead>
<tr>
<th>$NOMPOW$</th>
<th>$RPM &gt; \ REF$</th>
<th>$THOPT$</th>
<th>$FULL$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>$1 \rightarrow 0$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>$1 \rightarrow 0$</td>
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<tr>
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<td>$0 \rightarrow 1$</td>
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<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>$0 \rightarrow 1$</td>
</tr>
</tbody>
</table>

Table 3.1: Functionality of the logic control structure selector

The conditions for switching from power maximization to power control are that $NOMPOW = 1$ and $RPM > \ REF = 1$. The power maximization control system reduces generator speed by increasing power reference. Therefore, the before mentioned condition is an indication that the control signal produced by power maximization paradigm will be larger than the reference power, which is undesirable. Then another control structure is engaged.

The condition for switching to power optimization control structure follows similar logic. The conditions are that $RPM > \ REF = 0$ and $THOPT = 1$. When power available
from wind is reducing the reduction in pitch angle is required to keep the extracted power constant. When the pitch angle approaches optimal value it means that wind turbine is approaching its maximal production. The switching is performed when pitch angle is close to optimal and further reduction of pitch angle is required to speed up the wind turbine. This switching logic also constrains pitch angle at given wind speed to values larger than optimal pitch angle. There are several reasons why that is beneficial. Firstly, it ensures uniqueness of the wind turbine operating points (otherwise it would be possible to obtain the same power output at two different pitch angles). Secondly, larger pitch angles are more favorable in the case of sudden wind gust because the speed control is more responsive. Furthermore, this constraint ensures stability of the control system because at certain pitch angle that is necessarily smaller than optimal, the maximum of the $C_Q(\lambda, \beta)$ curve is reached. When this maximum is surpassed the partial derivative of aerodynamic torque with respect to pitch angle changes sign. This makes the speed controller that controls pitch angle unstable.

For this study it is important to notice that the switching of control structures may occur due to transients in the wind turbine in spite of obeying the condition $P_{\text{ref}} < P_{\text{max}}$. This can occur for instance due to large jumps in power reference. One possible remedy that often eliminates this kind of behavior is introduction of a rate limiter to the power reference.

### 3.2. NREL wind turbine control system

In comparison to V80 control system, the NREL control system is much simpler. Unlike V80 control system it does not use wind speed measurements and does not include additional blocks for the oscillation damping. Its inputs are the power demand and the generator speed measurement and the outputs are pitch angle reference and generator power reference. The controller structure is depicted on Fig. 3.8.

The NREL control system also consists of two tracking loops, the one which determines the power reference and the other which controls the pitch angle. There are two control configurations:

1. **Tracking the demanded power:**
   The reference power is equal to the demanded power increased to compensate for generator efficiency and limited to the generator rated power. The rotational speed of the wind turbine generator is controlled by a gain scheduled PI regulator. The scheduling is done according to the demanded power and reference pitch angle.

2. **Maximizing power production:**
   The reference power is determined according to a nonlinear function implemented as a look-up table. The pitch angle is set to zero.

The switching logic is such that the first configuration – the tracking of the demanded power – is active when at least one of the following conditions is satisfied:

1. $P_{\text{dem}} \leq T_{\text{opt}} \cdot \omega_g^{\text{filt}}$,
2. $\omega_g^{\text{filt}} \geq 121.6805 \text{ rad/s}$, or
3. $\beta_{\text{ref}}[k-1] > 0$.

The first condition is the most natural condition for the power tracking – it states that the available power is larger than the demanded power. The second condition engages pitching when the rated speed is (almost) reached – in order to start tracking the rated rotor speed. The third condition prevents jumps in reference pitch angle – active pitching has to drive the pitch angle to zero.

The optimal generator torque as function of generator speed is given on Fig. 3.2. As it can be seen from the figure, this characteristic consists of four parts: for very low generator speed the torque is zero (the wind turbine can not produce power if the rotor is rotating at very low speed), for low generator speed the characteristic is linear (affine), at medium generator speeds the characteristic is quadratic and at high generator speeds again linear (affine). The function is not defined for generator speeds above the nominal speed because at such wind speeds the optimal torque control is never active.
Figure 3.9: NREL torque characteristic

It is well known from wind turbine control theory that the rotor torque has to be proportional to the squared rotor speed to maximize the power output, see [1]. This optimal speed - torque curve can not be implemented in the whole operating region due to constraints on generator speed. The linear parts of the characteristic are suboptimal characteristics. The shape of this characteristic influences static behavior of the wind turbine, but also the dynamic behavior. Therefore, in the following sections operating points at different parts of this optimal curve will be treated as different operating regimes.
Chapter 4

Static model of the wind turbine

In this chapter the static models of the analyzed wind turbines will be presented. These models will be the basis for the derivation of linearized and piecewise-affine dynamics of the wind turbine.

As it was previously mentioned, only the wind turbines that are in power tracking operating regime are actual actuators in the wind farm. The wind turbines that maximize their power output are a disturbance for the wind farm control. For the NREL wind turbine all the operating regimes were modeled and analyzed. Such an overall model enables the modeling of the disturbances in wind farm. For the V80 wind turbine only the tracking of the demanded power will be described, because the modeling of the optimal power extraction regime would require further identification of undisclosed subsystems. Also, the conditions (static, as well as dynamic conditions that will be described in the following chapter) are identified that initiate the switching of the regimes. These constraints can be used to either design the control system that does not allow the switch to the power maximization mode or a better prediction of wind farm behavior.

4.1. Static model of the V80 wind turbine

According to the description of the wind turbine subsystems provided in the previous chapter, at the most generic level one can describe a power controlled wind turbine as a coupled system containing two control loops, see Fig. 4.1. First is generator speed control loop, which uses pitch mechanism as an actuator, and the second is power control loop that uses controllable power source to track electrical power reference. The speed reference is obtained from wind speed measurements and the power reference is user provided.

In the Sections 2.6 and 3.1.2 it was shown that the controllers from Fig. 4.1 contain integral behavior, which means that they eliminate steady state error. Also, Sec. 3 explains that if the power control structure is active that both references will be obtainable. Therefore,
Figure 4.1: The V80 wind turbine as a coupled control system

for power controlled wind turbine in steady state one can write:

\[ \Omega_g = \Omega_{\text{ref}}, \quad (4.1) \]
\[ P = P_{\text{dem}}, \quad (4.2) \]

The speed reference is defined by measured wind speed according to the relation (3.1).

The static model of wind turbine is composed from the aerodynamic conversion model (Sec. 2.1), the generator model (Sec. 2.2) and the model of drive train that contains only the gearbox and static losses (Sec. 2.3).

Both the model of aerodynamic conversion and the generator model are implemented in the simulation model as static relations. The drive train in steady state is described by following equations:

\[ \Omega_g = n_{gb} \cdot \Omega_r, \quad (4.3) \]
\[ T_r = n_{gb} \cdot T'_g, \quad (4.4) \]
\[ T'_g = C_{\omega g} \cdot \Omega_g + C_T g \cdot T_g \quad (4.5) \]

The inputs to the static model are wind speed, \( V \), and power reference, \( P_{\text{ref}} \). The obtained static model of power controlled wind turbine is:

\[ \Omega_g = \Omega_{\text{ref}}, \quad (4.6) \]
\[ \Omega_r = \frac{\Omega_{\text{ref}}}{n_{gb}}, \quad (4.7) \]
\[ P = P_{\text{ref}}, \quad (4.8) \]
\[ T_g = (P_{\text{ref}} + P_{\text{loss}}) \cdot \frac{K_{PT} \cdot \Omega_S}{p \cdot \Omega_{\text{ref}}}, \quad (4.9) \]
\[ T_r = n_{gb} \cdot \left( C_{\omega g} \cdot \Omega_{\text{ref}} + C_T g \cdot (P_{\text{ref}} + P_{\text{loss}}) \cdot \frac{K_{PT} \cdot \Omega_S}{p \cdot \Omega_{\text{ref}}} \right). \quad (4.10) \]

These expressions determine all wind turbine states except pitch angle. Steady state pitch angle is a solution to the equation:

\[ \frac{\pi}{2} \rho R^2 v^2 C_Q \left( \frac{\Omega_{\text{ref}} \cdot R}{n_{gb} \cdot V} \cdot \beta \right) = n_{gb} \cdot \left( C_{\omega g} \cdot \Omega_{\text{ref}} + C_T g \cdot (P_{\text{ref}} + P_{\text{loss}}) \cdot \frac{K_{PT} \cdot \Omega_S}{p \cdot \Omega_{\text{ref}}} \right). \quad (4.11) \]
In all admissible operating regimes (4.11) has at least one solution (otherwise provided speed and power references would not be obtainable). However, the solution is not necessarily unique because $C_Q(\cdot, \beta)$ is a nonlinear function. Function $C_Q(\cdot, \beta)$ has, as well as power coefficient, a distinct maximum and therefore two regions with opposite slopes. The slope of torque coefficient also determines the slope of aerodynamic torque. The speed regulation in this control structure rests on the fact that wind turbine is slowed down by increase of pitch angle. Therefore, the operating point should always be on the part of the $C_Q(\cdot, \beta)$ curve with the negative slope. The logic implemented in wind turbine controller ensures this, see Sec. 3.1.5.

It is interesting to notice here which states of the derived model are effected by power reference. One observes that wind turbine and generator speeds can not be effected by power references. The change in electrical power output as result of change of power reference is accomplished by changing torques on the shaft – at generator side by changing rotor current, and on wind turbine side by changing pitch angle. Additional interdependencies will be introduced by including dynamics into the model.

Further, the static condition for a wind turbine to be power controlled is derived. It is a condition that constraints reference power by a nonlinear function in wind speed:

$$P_{\text{ref}} \leq P_{\text{max}}(V).$$  

(4.12)

The function $P_{\text{max}}(V)$ is defined as:

$$P_{\text{max}}(V) = \left( \frac{P_{a,\text{max}}(V)}{\Omega_{\text{ref}}(V)} - C_{w_k} \cdot \Omega_{\text{ref}}(V) \right) \cdot \frac{p \cdot \Omega_{\text{ref}}}{C_{T_\theta} \cdot K_{PT} \cdot \Omega_{S}} - P_{\text{loss}},$$  

(4.13)

and

$$P_{a,\text{max}}(V) = \frac{\pi}{2} \rho R^2 V^3 \max(\beta, \lambda(V)).$$  

(4.14)

The $C_P(\beta)$ function has a distinctive and finite maximum, as the consequence of aerodynamic features of the blade, \[1\]. The derived equations determine a unique upper bound on power reference that is a function only in wind speed. Fig. 4.2 graphically depicts the derived constraint.

On the following figures the derived relations (4.6)–(4.11) are graphically described. The characteristics are plotted only in the regions where given operating conditions satisfy (4.12).

In Fig. 4.3 the operating region for power control of wind turbine can clearly be seen. The constraints visible on this figure correspond to characteristic provided in Fig. 4.2. In Fig. 4.5 the effect of the stationary speed control is visible. Rotor speed depends solely on the wind speed and changes of the operating point by providing different power references can not statically influence rotational speed.

Fig. 4.4 demonstrates conceptual similarity between power control and torque control. The change in power at constant wind speed is achieved by changing the torque proportionally to reference power. The proportionality coefficient depends on generator speed at given wind. As long as the rotational speed is constant, constant reference power produces constant torque
regardless of changes in wind speed. To produce constant aerodynamical torque at different wind speeds the blades are pitched. This can be seen in Fig. 4.6.

In Figure 4.7 a potential measures of fatigue load that should be minimized by power control is shown. Tower bending moment is proportional to thrust force, which is a function of wind speed, pitch angle and rotational speed. Increase in wind speed increases thrust force and increase in pitch angle reduces the thrust force. However, as it can be seen in Fig. 4.7, when wind turbine is tracking the constant power reference increase in wind speed reduces thrust force. This is due to the fact that increase in wind speed is followed by increase in pitch angle. Jointly, the influence of pitch angle on thrust overcomes the influence of wind speed increase and the thrust force is reduced. At constant wind, producing larger amount of power means reducing pitch angle, which increases rotor thrust and tower bending moment.
Figure 4.3: Produced electrical power

Figure 4.4: Aerodynamic torque
Figure 4.5: Generator speed

Figure 4.6: Pitch angle
4.2. Static model of the NREL wind turbine

The largest difference between the static model of the V80 and NREL wind turbine is that NREL wind turbine does not use wind measurements. The V80 wind turbine determines its speed reference based solely on wind speed measurements, while the rotational speed of the NREL wind turbine is determined implicitly.

As it was described in Ch. 3.2 and depicted in Fig. 3.8, the control system of the NREL wind turbine has two alternative configurations – power (and nominal speed) tracking with active pitch control, and power maximization configuration that keeps pitch angle at zero and controls the generator torque by a gain scheduled proportional regulator acting on generator speed feedback.

The static model of the NREL wind turbine generator and its control system is:

\[ T_g = \frac{P_{\text{ref}}}{\Omega_g}, \]  

\[ P = \mu T_g \Omega_g, \]  

where:

\( \mu \) is generator efficiency.

The drive train model of the NREL wind turbine does not describe the losses. Therefore, its static model is simply:

\[ T_r = n_{gb} T_g, \quad \Omega_g = n_{gb} \Omega_r. \]  

The power tracking control configuration is similar to the one described for V80 wind
turbine. Its static model is determined by the following equations:

\[
P_{\text{ref}} = \frac{1}{\mu_{\text{ctrl}}} P_{\text{dem}},
\]

\[
\Omega_g = \Omega_{\text{nom}},
\]

\[
\frac{1}{\mu_{\text{ctrl}}} \frac{P_{\text{dem}}}{\Omega_r} = \frac{1}{2} \rho R^3 \pi V^2 C_Q \left( \frac{\Omega_r R}{V}, \beta \right),
\]

where:

\( \mu_{\text{ctrl}} \) is the compensation for generator efficiency implemented in the controller.

The equation (4.20) implicitly determines the stationary pitch angle. This equation has two solutions and, as it was explained in the previous section, the stabile one is the one for which the \( C_Q(\lambda, \beta) \) has negative slope.

The power maximization control configuration is determined by the following equations:

\[
\beta = 0,
\]

\[
T^\text{ref}_g = \begin{cases} 
0, & \Omega_g \leq \Omega_g^{\text{min}} \\
 a_1 \cdot \Omega_g + b_1, & \Omega_g^{\text{min}} \leq \Omega_g \leq \Omega_g^{\text{opt}} \\
 K_{\text{opt}} \Omega_g^2, & \Omega_g^{\text{opt}} \leq \Omega_g \leq \Omega_g^{\text{Opt}} \\
 a_2 \cdot \Omega_g + b_2, & \Omega_g^{\text{Opt}} \leq \Omega_g \leq \Omega_g^{\text{nom}}
\end{cases}
\]

\[
P_{\text{ref}} = T^\text{ref}_g \cdot \Omega_g.
\]

The equation (4.22) describes the optimal torque characteristic depicted in Fig. 3.2. The introduced parameters can be read from the figure.

The operating point of the model can be determined by solving the equation:

\[
T_r = n_{gb} \cdot T^\text{ref}_g,
\]

where \( T_r \) is the aerodynamic torque determined by the expression:

\[
T_r = \frac{1}{2} \rho R^3 \pi V^2 C_Q \left( \frac{\Omega_g R}{n_{gb} V}, 0 \right).
\]

Since in this case not only the aerodynamic torque changes due to changes in rotor speed, the condition for stability of the operating point becomes more complicated. Here we use the first order model of the shaft to derive the stability condition. The first order model of the shaft is:

\[
T_r - n_{gb} \cdot T_g = J \frac{d\omega_r}{dt}.
\]

By linearizing this equation, we obtain a simple condition for system stability for changes in \( \omega_r \) at fixed \( \beta \):

\[
\left. \frac{\partial T_r}{\partial \omega_r} \right|_{\omega_r, \beta} \leq n_{gb} \left. \frac{\partial T^\text{ref}_g}{\partial \omega_r} \right|_{\omega_r, \beta}.
\]
This condition will be used to determine which of the solutions to (4.24) is the actual operating point of the system. The system is designed in such a way that the operating point for certain inputs is unique.

There is one more special case that needs to be considered. Let us consider the case when wind turbine cannot produce the demanded power at nominal speed. However, this does not mean that its maximal power is smaller than the demanded power, because the optimal tip-speed ratio is obtained at different rotor speed. Therefore, the condition for power control will be satisfied (optimal torque times generator speed will be larger than the demanded power), but in order to produce the demanded power the generator speed will need to be smaller than the rated speed. This means that the controller that determines the pitch angle will be saturated in zero. The governing static equations in this case will be:

\[
P_{\text{ref}} = \frac{1}{\mu_{\text{ctrl}}} P_{\text{dem}}, \quad (4.28)
\]

\[
\beta = 0, \quad (4.29)
\]

\[
T_r = n_{\text{gb}} \cdot T_{\text{gref}}. \quad (4.30)
\]

In Alg. 4.1 the algorithm for computing the unique operating point for given inputs to the wind turbine is provided.

**Algorithm 4.1** (Static model).

- **INPUT** $V_0$, $P_{\text{dem0}}$
- **OUTPUT** $\beta_0$, $\Omega_{r0}$, $T_{g0}$

**LET** $\Omega_r^0 = \omega_{\text{nom}}^0$, $\beta_0 = 0$, $P'_{\text{max}} = P_{\text{max}}(\frac{\Omega_{\text{nom}} R}{V_0}, \beta_0)$

**IF** $P_{\text{dem0}} \geq P'_{\text{max}}$ **power tracking mode**

**LET** $\beta_0 = \left\{ \beta \left| \frac{1}{2} \rho R^3 \pi V_0^2 C_Q(\frac{\Omega_{\text{nom}} R}{V_0}, \beta) = \frac{P_{\text{dem0}}}{\mu_{\text{ctrl}} n_{\text{gb}}}, \quad \frac{\partial C_Q(\frac{\Omega_{\text{nom}} R}{V_0}, \beta)}{\partial \beta} \leq 0 \right. \right\}$

**LET** $T_{g0} = \frac{P_{\text{dem0}}}{\mu_{\text{ctrl}} n_{\text{gb}}}$

**ELSE**

**power maximization**

$C_P^{\text{opt}} = \max(C_P(\frac{\omega R}{V_0}, \beta_0))$

**LET** $\Omega_{r0} = \arg \max_{\omega_r} C_P(\frac{\omega R}{V_0}, \beta_0)$

**IF** $\Omega_{r0} \cdot n_{\text{gb}} \leq \Omega_{\text{opt}}^{\text{affine}}$

**affine optimal torque characteristic**

**LET** $\Omega_{r0} = \left\{ \omega_r \left| \frac{1}{2} \rho R^3 \pi V_0^2 C_Q(\frac{\omega R}{V_0}, \beta_0) = a_1 n_{\text{gb}} \omega_r + b_1, \quad \frac{\partial C_Q(\frac{\omega R}{V_0}, \beta_0)}{\partial \omega_r} \leq 0 \right. \right\}$

**LET** $T_{g0} = a_1 n_{\text{gb}} \Omega_{r0} + b_1$
ELSE IF $\Omega_{n_0} \cdot n_{gb} \geq \Omega_{g}^{Opt}$

affine optimal torque characteristic – upper part $- \lambda < \lambda_{opt}$

LET $\Omega_{r_0} \leftarrow \left\{ \omega_r \middle| \frac{1}{2} \rho R^2 \pi V_0^2 C_P(\frac{\omega R}{V_0}, \beta_0) = a_3 n_{gb} \omega_r + b_3, \quad \frac{\partial C_Q(\frac{\omega R}{V_0}, \beta_0)}{\partial \omega_r} \leq a_3 n_{gb} \frac{2}{\rho \pi R^2 V_0} \right\}$

LET $T_{g_0} \leftarrow a_3 n_{gb} \Omega_{r_0} + b_3$

ELSE

quadratic optimal torque characteristic

LET $T_{g_0} \leftarrow K_{opt} (n_{gb} \Omega_{r_0})^2$

END

IF $\frac{1}{2} \rho R^2 \pi V_0^2 C_P(\frac{\Omega_{n_0} R}{V_0}, \beta_0) > \frac{P_{\text{limit}}}{\mu}$

maximal power (at rotating speed smaller than nominal) larger than demanded power

LET $\Omega_{r_0} \leftarrow \left\{ \omega_r \middle| \frac{1}{2} \rho R^2 \pi V_0^2 C_P(\frac{\omega R}{V_0}, \beta_0) = \frac{P_{\text{limit}}}{\mu}, \quad \frac{\partial C_Q(\frac{\omega R}{V_0}, \beta_0)}{\partial \omega_r} \leq - \frac{P_{\text{limit}}}{(\omega_r n_{gb})^2 \rho \pi R^2 V_0} \right\}$

LET $T_{g_0} \leftarrow \frac{P_{\text{limit}}}{\mu n_{gb} \omega_r}$

END

END

The following figures present the obtained static model.

The figures 4.8 and 4.9 clearly depict the two control configurations. The figure 4.10 shows that different behavior of rotational speed in different areas of torque characteristic. One detail of the dependency of wind turbine states on wind speed for the demanded power 1.5 MW is given on figure 4.13. At this figure the previously mentioned operating region in which the pitch controller is saturated is denoted by dotted red lines.

The figure 4.11 that depicts the thrust force is specially interesting. It shows that the wind turbine experiences the largest thrust force when it transfers from maximization of power production to power limitation. This is because the increase in rotational speed increases the thrust force and increase in pitch angle reduces the thrust force, and the edge between the two control configurations is the operating region with largest rotational speed at smallest pitch angle. This means that if one wishes to control the wind farm to minimize the thrust, one should run all the wind turbines at as large pitch angle as possible. The pitch angle can be actively increased by reducing the demanded power. Also, if two wind turbines are given the same power demand, larger thrust force will be experienced by the wind turbine with smaller wind speed. On the other hand, the figure 4.12 shows that the shaft moment and the produced power plots have exactly the same shape.
Figure 4.8: Produced electrical power.

Figure 4.9: Pitch angle.
Figure 4.10: Generator speed.

Figure 4.11: Thrust force.
Figure 4.12: Shaft moment.

Figure 4.13: Wind turbine states for $P_{\text{dem}} = 1.5$ MW
Chapter 5

Dynamical model of the wind turbine

In this chapter the dynamic models of the V80 and NREL wind turbines will be presented. The full dynamical model of the wind turbine that is available describes wind turbine dynamics in more details that it is necessary for wind farm control. The phenomena such as shaft oscillations or tower oscillations are too fast to be actively controlled at wind farm level. Also, an attempt to control such behavior would cause collision with wind turbine local control system. Therefore, as a higher level of control hierarchy, the reconfigurable wind farm controller needs the model that describes the slower wind turbine dynamics. Therefore, in this chapter the fast dynamics will be eliminated from the wind turbine models. The wind turbine dynamics will be linearized at different operating regimes that were described in Ch. 4. The models derived in this chapter along with static models from Ch. 4 describe the wind turbine models in any admissible operating point of the NREL wind turbine and every operating point at which the wind turbine delivers demanded power for V80 wind turbine. These models also allow construction of piecewise-affine model of the wind turbine with complete freedom in choice of regions on which different affine dynamics will be defined.

5.1. Dynamical model of the V80 wind turbine

The dynamic model of power controlled wind turbine is easily obtained by combining the subsystems described in Ch. 2. The model obtained in that manner is a complex nonlinear model. Since more than few subsystems described in previous chapter are identified models, certain error might be accumulated by connecting together subsystems that have even a minor error in comparison to subsystems of the original model. The following figures are results of parallel simulation of identified and original model with inputs that cover the entire operating region in which wind turbine is power controlled. The inputs to the model are shown in Fig. 5.1. Figures 5.2–5.5 show wind turbine states and outputs.

From Figures 5.2–5.5 it can be seen that errors in wind turbine states and outputs are negligible. This confirms that models of wind turbine’s subsystems were well identified and
Figure 5.1: Model inputs

Figure 5.2: Speed reference and pitch angle
Figure 5.3: Wind turbine and generator speed

Figure 5.4: Wind turbine rotor and generator torque
that modeling errors are not significant.

Regardless of its accuracy, the dynamic system obtained by combining the described subsystems is not adequate for reconfigurable control design. Firstly, it contains too fast dynamics for wind farm control. Wind farm control system should not compete with wind turbine local control. However, it has to act at time scales relevant for wind farm disturbances, for example strong wind gusts. Therefore, all fast transients in the model should be omitted.

As a first step the sources of high frequency oscillations were removed - tower nodding, tower side-side motion and shaft oscillations. Removing the tower nodding did not influence the model response much because local speed controller provides significant damping to tower oscillations. The shaft oscillations too are significantly damped by interventions in the control system. The tower side-side motion has minor effect on wind turbine operation.

Removing shaft oscillations and tower side-side motion required the revision of drive train model. The drive train was modeled as a shaft with lumped inertia, gearbox and drive-train losses. The obtained model is:

\[
\Omega_t(s) = \frac{1}{J_t \cdot s} \left(T_t(s) - n_{gb} \cdot C_T \cdot T_g(s) \right),
\]

\[
\Omega_g(s) = n_{gb} \cdot \Omega_t(s),
\]

where:

\[J_t = J_t + n_{gb}^2 \cdot J_g\] is lumped wind turbine inertia.
In Fig. 5.6 the frequency characteristic of the original transmission system and the approximation is provided. It can be seen that this approximation characterizes the system perfectly at low frequencies. The difference occurs at higher frequencies, mostly higher than 1 Hz, which is approximately the control frequency of reconfiguration control. Therefore, it is presumable that this approximation will not induce large modeling errors.

With elimination of drive train oscillations the transmission filter in the generator speed feedback was made unnecessary, so it was left out of the model. The second intervention to the control system used to reduce oscillations, the UMPDAMP subsystem could not be simply eliminated because this filter is, unlike Transmission filter, effective at all frequencies. It was simplified as a part of generator power control system, as it will be shown later.

The next intervention was assuming pitch control and positioning system linear and infinitely fast. This is justified because the pitch drive dynamics is reasonably faster than wind turbine dominant dynamics (rotor dynamics). Those model simplification produced negligible effects on wind turbine state response, too.

Another source of very fast model dynamics is the model of generator power control. This system contains complex dynamics, mainly due to the extension for shaft oscillation damping. The generator model is a nonlinear model. To perform model simplification, the generator model was linearized at an operating point determined by power reference $P_{\text{ref}}$ and wind speed $V_0$. Following transfer functions were obtained:

\[
\frac{\mathbf{T}_g(s)}{\mathbf{P}_{\text{ref}}(s)} = \frac{K_{\text{PT}} \cdot \frac{\Omega_0}{\mathbf{p} \Omega_0}}{\mathbf{T}_{\text{gcl}s} + 1}, \quad (5.3)
\]

\[
\frac{\mathbf{T}_g(s)}{\Omega_g(s)} = K_{\text{PT}} P_{\text{stat0}} \cdot \frac{\left( T_{\text{gcl}} K^D \frac{\Omega_0}{P_{\text{stat0}}} - T_1^D T_2^D \right) s^2 - (T_1^D + T_2^D)s - 1}{(T_{\text{gcl}s} + 1)(T_1^D s + 1)(T_2^D s + 1)}, \quad (5.4)
\]

\[
\frac{\mathbf{P}(s)}{\Omega_g(s)} = \frac{1}{\mathbf{T}_{\text{gcl}s} + 1}, \quad (5.5)
\]

\[
\frac{\mathbf{P}(s)}{\mathbf{\Omega}_g(s)} = \frac{p}{\mathbf{\Omega}_g} \cdot \left( \frac{\Omega_0}{K_{\text{PT}}} \cdot \frac{T_g(s)}{\mathbf{\Omega}_g(s)} + P_{\text{stat0}} \right), \quad (5.6)
\]

where:

$T_{\text{gcl}} = \frac{\mathbf{T}}{K_{\text{PT}}^{\mathbf{p}}}, \quad \frac{\Omega_0}{\mathbf{p} \Omega_0}$ is generator power control time constant, $T_{\text{gcl}} \in [0.0995 \, \text{s}, 0.1632 \, \text{s}]$, $\Omega_0$ is generator speed at the operating point, $P_{\text{stat0}} = (P_{\text{dem0}} + P_{\text{loss}}) \cdot \frac{\Omega_0}{\mathbf{p} \Omega_0}$ is the stator power at operating point.

These transfer functions were simplified focusing only on their behavior at low frequencies. Since the time constant of the generator control loop is significantly smaller than target frequency range for wind farm control, generator control was considered infinitely fast and
Figure 5.6: Frequency characteristics of wind turbine drive train
the following transfer functions were derived:

\[
\frac{T_g(s)}{P_{ref}(s)} = K_{PTg}, \quad (5.7)
\]

\[
\frac{T_g(s)}{\Omega_g(s)} = K_{\Omega Tg}, \quad (5.8)
\]

\[
\frac{P(s)}{P_{ref}(s)} = 1, \quad (5.9)
\]

\[
\frac{P(s)}{\Omega_g(s)} = 0, \quad (5.10)
\]

where:

\[
K_{PTg} = \frac{K_{PTg} \Omega_s}{P_{dem0}}, \quad \text{and}
\]

\[
K_{\Omega Tg} = K_{PT} \cdot \frac{\Omega_s}{P_{dem0}} \cdot (P_{dem0} + P_{loss}).
\]

Further model simplifications were made by linearizing nonlinear static models of wind turbine aerodynamics. Linear relation for aerodynamic torque is:

\[
T_r(s) = K_{\Omega T} \Omega_r(s) + K_{\beta T} \beta(s) + K_{V T} V(s), \quad (5.12)
\]

and the relation for thrust force:

\[
F_T(s) = K_{\Omega F} \Omega (s) + K_{\beta F} \beta(s) + K_{V F} V(s). \quad (5.13)
\]

Finally, the simplifications were made to wind turbine local controller. Gain scheduled controller was substituted by a constant gain controller. The controller gains were fixed at their values at the operating point. The generator speed measurement was assumed perfect.

After performing all mentioned simplifications the wind turbine model comes down to two different formulations of linear dynamic model at an operating point. For wind speeds between \(v_{\omega_{\text{min}}} = 4.97 \text{ m/s}\) and \(v_{\omega_{\text{max}}} = 8.15 \text{ m/s}\) it is a linear model with 5 states. For wind speeds outside this range it is a linear model with 2 states. The source of duality of the model is saturation of generator speed reference outside the mentioned region, i.e. generator speed reference is constant. Generation of speed reference contains the cascade of several filters with time constants in relevant range for reconfigurable control. First, wind speed is filtered during measuring. Then, wind speed measurement are filtered inside \textit{Stationary control} block to compute generator speed reference. Finally, speed reference is filtered inside the speed controller. All of these filters were approximated by first order linear filters of time constants 1 s, 2 s and 8 s. The two common states of the models are related to speed controller and wind turbine inertia.

Further, the matrices of the state space models for the two dynamic models will be given. The state space matrices are defined according to:

\[
\dot{x} = A \cdot x + B \cdot u + B_d \cdot d. \quad (5.14)
\]
Both models have same input and disturbance vectors.

\[ u = [P_{ref}], \quad \text{and} \]
\[ d = [v]. \quad (5.15) \]
\[ d = [v]. \quad (5.16) \]

For \( v \in [v_{\omega_{min}}, v_{\omega_{max}}] \) system state is defined as

\[ x = \begin{bmatrix} \beta \\ \omega_r \\ \omega_{ref} \\ v_{meas} \end{bmatrix}, \quad (5.17) \]

and the state space matrices are:

\[ A = \begin{bmatrix} \frac{-30 \pi K_{Pn} g_b \cdot K_{T_r}}{J_t} & -\frac{30 \pi K_{Pn} g_b \cdot \left(K_{I} + \frac{K_{I} T_r - n_{gb}^2 C_{T_g} K_{\Omega r} T_r}{J_t}\right)}{J_t} & K_{P} K_{I} - \frac{K_{P}}{T_{\Omega f}} & \frac{K_{P}}{T_{\Omega f}} & 0 \\ \frac{K_{\beta T_e}}{J_t} & 0 & -\frac{1}{T_{\Omega f}} & 0 & 0 \\ 0 & 0 & -\frac{1}{T_{sc}} & 0 & -\frac{k}{T_{em}} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (5.18) \]

\[ B = \begin{bmatrix} \frac{30 \pi K_{Pn} g_b \cdot C_{T_g} K_{P T_g}}{J_t} \\ -n_{gb} C_{T_g} K_{P T_g} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad (5.19) \]

\[ B_d = \begin{bmatrix} -\frac{30 \pi K_{Pn} g_b K_{v T_r}}{J_t} \\ \frac{K_{v T_r}}{J_t} \\ 0 \\ 0 \\ \frac{1}{T_{vm}} \end{bmatrix}. \quad (5.20) \]

For wind speeds \( v \in [v_{cut in}, v_{\omega_{min}}] \cup v \in [v_{\omega_{max}}, v_{cut out}] \) the system state is defined as:

\[ x = \begin{bmatrix} \beta \\ \omega_r \end{bmatrix}, \quad (5.21) \]
and the state space matrices are:

\[
A = \begin{bmatrix}
-\frac{30}{\pi} \cdot n_{gb} \cdot K_P \cdot \frac{K_{Pr} - J_t}{J_t} & -\frac{30}{\pi} \cdot n_{gb} \cdot K_P \cdot \left( K_I + \frac{K_{Gt} - n_{gb}^2 C_{Tg} K_{Gt}}{J_t} \right)
\end{bmatrix},
\]

(5.22)

\[
B = \begin{bmatrix}
\frac{30}{\pi} \cdot n_{gb}^2 K_P \cdot \frac{C_{Tg} K_{Pr}}{J_t}
\end{bmatrix},
\]

(5.23)

\[
B_d = \begin{bmatrix}
-\frac{30}{\pi} n_{gb} K_P \cdot \frac{K_{cTg}}{J_t}
\end{bmatrix}
\]

(5.24)

The following figures show simulation results that compare the response of the original model and the obtained approximation. The outputs of the model are linear combinations of its states, according to relations (5.2), (5.12), (5.13) and (5.7)–(5.10). The simulations are performed at three different operating points, for:

- high wind: \( v_0 = 20 \text{ m/s} \), \( P_{dem0} = 1800 \text{ kW} \),
- moderate wind: \( v_0 = 12 \text{ m/s} \), \( P_{dem0} = 1200 \text{ kW} \), and
- low wind: \( v_0 = 6 \text{ m/s} \), \( P_{dem0} = 200 \text{ kW} \).

The first two operating points fall into the operating region with constant speed reference and therefore are approximated with second order model. Around the third operating point speed reference is variable and the approximation was made by the fifth order model.
Figure 5.7: Model inputs, $v_0 = 20 \text{ m/s}$, $P_{dem0} = 1800 \text{ kW}$

Figure 5.8: Speed reference and pitch angle, $v_0 = 20 \text{ m/s}$, $P_{dem0} = 1800 \text{ kW}$
Figure 5.9: Wind turbine and generator speed, $v_0 = 20 \text{ m/s}$, $P_{dem0} = 1800 \text{ kW}$

Figure 5.10: Wind turbine rotor and generator torque, $v_0 = 20 \text{ m/s}$, $P_{dem0} = 1800 \text{ kW}$
Figure 5.11: Model outputs - electrical power and thrust force, $v_0 = 20 \text{ m/s}$, $P_{dem0} = 1800 \text{ kW}$

Figure 5.12: Model inputs, $v_0 = 12 \text{ m/s}$, $P_{dem0} = 1200 \text{ kW}$
Figure 5.13: Speed reference and pitch angle, $v_0 = 12 \text{ m/s}$, $P_{dem0} = 1200 \text{ kW}$

Figure 5.14: Wind turbine and generator speed, $v_0 = 12 \text{ m/s}$, $P_{dem0} = 1200 \text{ kW}$
Figure 5.15: Wind turbine rotor and generator torque, $v_0 = 12$ m/s, $P_{dem0} = 1200$ kW

Figure 5.16: Model outputs - electrical power and thrust force, $v_0 = 12$ m/s, $P_{dem0} = 1200$ kW
Figure 5.17: Model inputs, \( v_0 = 6 \text{ m/s}, \ P_{\text{dem}0} = 200 \text{ kW} \)

Figure 5.18: Speed reference and pitch angle, \( v_0 = 6 \text{ m/s}, \ P_{\text{dem}0} = 200 \text{ kW} \)
Figure 5.19: Wind turbine and generator speed, $v_0 = 6 \text{ m/s}$, $P_{dem0} = 200 \text{ kW}$

Figure 5.20: Wind turbine rotor and generator torque, $v_0 = 6 \text{ m/s}$, $P_{dem0} = 200 \text{ kW}$
The simulation results in Figures 5.7–5.21 generally show that the responses of the derived approximative model and the original model coincide very good. The aim of the model approximation was to derive as simple system model as possible that still describes system behavior at time scale with order of magnitude of 1 s. The obtained simulation results are from this perspective more than satisfactory. The obtained model has very low order. For major part of operating region, and the part that is more interesting for power control since the available power is higher, the model is of second order.

Naturally, the approximations and especially linearization introduce some error, especially for large excursions from operating point. In the simulations the turbulent wind with large standard deviation was used and the power references were deviating from operating point relatively large. Still, no major error was accumulated in the responses and all dynamical development on relevant time scale was captured by the approximated model.

One surprising thing about the derived model is complete lack of dynamics in the model of electrical power. From the simulations it can be observed that this is justified because the changes in power reference are tracked very fast by the generator power control and the influence of other system disturbances on electrical power output is minor. The reason for this lies in a very fast generator power control loop that almost momentarily compensates for all disturbances in the system by adjusting stator power.
5.2. Dynamical model of the NREL wind turbine

The dynamical model of the NREL wind turbine is more straightforward than the V80 dynamical model, mainly because there is no additional oscillation damping control system. The models of the dynamical subsystems of this model are described in Ch. 2 and 3.

The dominant dynamics at relevant frequencies arise from the system’s rotational inertia and speed control. Also, the speed controller uses filtered measured generator speed. The time constant of this filter is also relevant for the overall system behavior. The simplifications of tower dynamics and shaft oscillations were performed in the same manner as it is described for the V80 wind turbine in the previous section.

The following notation will be used:

\( J_g, J_r \) are inertias of the rotor and the generator,
\( J \) is the equivalent inertia for the simplified shaft model, \( J = J_r + n^2 g J_g \),
\( K_P, K_I, K_{corr} \) are parameters of the pitch controller, where \( K_P \) and \( K_I \) are fixed parameters and \( K_{corr} \) depends on operating point, see Fig. 3.8, and
\( P_{dem0}, \Omega_0, V_0 \) are demanded power, wind speed and (generator/rotor) speed in the operating point.

The partial derivatives of the thrust force and aerodynamic torque are given by:

\[
K_vT_r = \left. \frac{\partial T_r}{\partial v} \right|_{\Omega_0, \beta_0, V_0} = \frac{1}{2} \rho \pi R^2 (C_Q(\lambda_0, \beta_0)) \cdot 2 \cdot V_0 - \Omega_0 R \cdot \left. \frac{\partial C_Q(\lambda, \beta)}{\partial \lambda} \right|_{\lambda_0, \beta_0} \tag{5.25}
\]

\[
K_\beta T_r = \left. \frac{\partial T_r}{\partial \beta} \right|_{\Omega_0, \beta_0, V_0} = \frac{1}{2} \rho \pi R^2 V_0^2 \left. \frac{\partial C_Q(\lambda, \beta)}{\partial \lambda} \right|_{\lambda_0, \beta_0} \tag{5.26}
\]

\[
K_\omega T_r = \left. \frac{\partial T_r}{\partial \omega_r} \right|_{\Omega_0, \beta_0, V_0} = \frac{1}{2} \rho \pi R^2 V_0^2 \left. \frac{\partial C_Q(\lambda, \beta)}{\partial \lambda} \right|_{\lambda_0, \beta_0} \tag{5.27}
\]

\[
K_vF_T = \left. \frac{\partial F_T}{\partial v} \right|_{\Omega_0, \beta_0, V_0} = \frac{1}{2} \rho \pi R^2 (C_T(\lambda_0, \beta_0)) \cdot 2 \cdot V_0 - \Omega_0 R \cdot \left. \frac{\partial C_T(\lambda, \beta)}{\partial \lambda} \right|_{\lambda_0, \beta_0} \tag{5.28}
\]

\[
K_\beta F_T = \left. \frac{\partial F_T}{\partial \beta} \right|_{\Omega_0, \beta_0, V_0} = \frac{1}{2} \rho \pi R^2 V_0^2 \left. \frac{\partial C_T(\lambda, \beta)}{\partial \lambda} \right|_{\lambda_0, \beta_0} \tag{5.29}
\]

\[
K_\omega F_T = \left. \frac{\partial F_T}{\partial \omega_r} \right|_{\Omega_0, \beta_0, V_0} = \frac{1}{2} \rho \pi R^2 V_0^2 \left. \frac{\partial C_T(\lambda, \beta)}{\partial \lambda} \right|_{\lambda_0, \beta_0} \tag{5.30}
\]

There are 4 different dynamical models developed, each contributed to a certain operating region. The dynamical model of the system will be presented in the state-space form defined as:

\[
\dot{x} = A \cdot x + B \cdot u + B_d \cdot d, \quad \tag{5.31}
\]

\[
y = C \cdot x + D \cdot u + D_d \cdot d, \quad \tag{5.32}
\]

where:
\( x \) is the state vector,
\( u \) is the control input,
\( d \) is the disturbance input, and
\( y \) is the system output.

The states, inputs and outputs of the model are sometimes defined differently for different operating regions.

### 5.2.1. Power controlled wind turbine

In this region the demanded power is produced and the rotational speed is controlled by pitching the blades. The equation that describes power production of the wind turbine at time scale of 1 s is simply:

\[
P = P_{\text{dem}}. \tag{5.33}
\]

This relation is a consequence of extremely fast and unconstrained generator control.

The relevant dynamics of this operating region is described by the state-space system where vectors are defined as:

\[
x = \begin{bmatrix} \beta \\ \omega_r \\ \omega_{\text{filt}} \end{bmatrix}, \tag{5.34}
\]

\[
u = \begin{bmatrix} P_{\text{dem}} \end{bmatrix}, \tag{5.35}
\]

\[
d = \begin{bmatrix} v \end{bmatrix}, \tag{5.36}
\]

\[
y = \begin{bmatrix} F_i \\ M_{\text{shaft}} \end{bmatrix}, \tag{5.37}
\]

\[
(5.38)
\]
and the state space matrices are:

\[
A = \begin{bmatrix}
0 & \frac{-K_{\omega} n_{gb}}{K_{corr} T_{c}} & \frac{K_{P}}{K_{corr} T_{c}} - \frac{K_{I}}{K_{corr}} \\
0 & \frac{1}{J_{gb} T_{c}} & 0 \\
\frac{1}{J_{gb} \mu} & \frac{1}{J_{gb} T_{c}} - \frac{1}{T_{c}} & 0 \\
\end{bmatrix},
\]

(5.39)

\[
B = \begin{bmatrix}
0 \\
\frac{-1}{J \mu} \\
0 \\
\end{bmatrix},
\]

(5.40)

\[
B_d = \begin{bmatrix}
0 \\
\frac{1}{J} \cdot K_{v T} \\
0 \\
\end{bmatrix},
\]

(5.41)

\[
C = \begin{bmatrix}
K_{\beta F_T} \\
\frac{K_{\beta F_T}}{\omega_{gb}} \cdot \frac{J_{gb}}{J} \cdot K_{\beta T} \\
\frac{K_{\omega F_T}}{n_{gb}^2} \cdot \frac{J_{gb}}{J} \cdot K_{\omega T} - \frac{n_{gb}}{J} \cdot \frac{P_{dem} \cdot n_{gb}}{J} \cdot \frac{P_{dem} \cdot n_{gb}}{\mu \omega_{gb}^2} \\
\end{bmatrix},
\]

(5.42)

\[
D = \begin{bmatrix}
0 \\
\frac{1}{J \mu} \\
0 \\
\end{bmatrix}
\]

(5.43)

\[
D_b = \begin{bmatrix}
K_{v F_T} \\
\frac{K_{v F_T}}{n_{gb}^2} \cdot \frac{J_{gb}}{J} \cdot K_{v T} \\
\end{bmatrix}
\]

(5.44)

The model was tested at operating point defined by \( P_{dem} = 4.5 \) MW and \( v = 20 \) m/s. The results are given in Fig.5.22.

### 5.2.2. Power maximization – optimal torque characteristic \( T_{ref}^g = K_{opt} \cdot \omega_{g}^2 \)

In this operating region the generator torque reference is determined based on the measurements of the generator speed. The pitch controller is inactive:

\[
\beta = 0.
\]

(5.45)

The power production in this operating region is determined solely on the available wind – the power demand provided by the operator has no influence on the power production. Therefore, the system has no control input.

The relevant dynamics of this operating region is described by the state-space system where vectors are defined as:

\[
x = \begin{bmatrix}
\omega_{r} \\
\omega_{filt} \\
\end{bmatrix},
\]

(5.46)

\[
d = \begin{bmatrix}
v \\
\end{bmatrix},
\]

(5.47)

\[
y = \begin{bmatrix}
P \\
F_t \\
M_{shaft} \\
\end{bmatrix},
\]

(5.48)

\[
(5.49)
\]
Figure 5.22: Simulation of the wind turbine in power tracking regime.
and the state space matrices are:

\[
A = \begin{bmatrix}
\frac{1}{\nu} \cdot K_{\omega T_r} & -\frac{1}{\nu} \cdot n_{gb} \cdot 2 \cdot K_{opt} \cdot \Omega g0 \\
\frac{n_{gb}}{\nu} & -\frac{1}{\nu}
\end{bmatrix},
\]

\[(5.50)\]

\[
B_d = \begin{bmatrix}
\frac{1}{\nu} K_{\omega T_r} \\
0
\end{bmatrix},
\]

\[(5.51)\]

\[
C = \begin{bmatrix}
K_{\omega F_T} \\
\frac{n_{gb}^2 \cdot \frac{J_g}{J} \cdot K_{\omega T_r}}{\nu} & 2 \cdot n_{gb} \cdot \frac{J}{J} \cdot K_{opt} \cdot \Omega g0
\end{bmatrix},
\]

\[(5.52)\]

\[
D_b = \begin{bmatrix}
K_{\omega F_T} \\
\frac{n_{gb}^2}{\nu} \cdot \frac{J}{J} \cdot K_{\omega T_r}
\end{bmatrix},
\]

\[(5.53)\]

The model was tested at operating point defined by \(P_{dem} = 5\) MW and \(v = 8.5\) m/s. The results are given in Fig.5.23.

5.2.3. Power maximization – affine torque characteristic \(T_{g}^{ref} = a \cdot \omega_g + b\)

This operating regime actually falls into the previously described operating region – power maximization. However, different form of control function leads to different state-space description. There are two distinct operating regions in which this type of control is used. Their only difference (from modeling perspective) is that they have different parameters \(a\) and \(b\). Analogously to the previous operating region, the pitch controller is inactive:

\[
\beta = 0,
\]

\[(5.54)\]

and the system has no control input.

The relevant dynamics of this operating region is described by the state-space system where vectors are defined as:

\[
x = \begin{bmatrix}
\omega_r \\
\omega_{filt}
\end{bmatrix},
\]

\[(5.55)\]

\[
d = \begin{bmatrix}
v
\end{bmatrix},
\]

\[(5.56)\]

\[
y = \begin{bmatrix}
P \\
F_t \\
M_{shaft}
\end{bmatrix},
\]

\[(5.57)\]
Figure 5.23: Simulation of the wind turbine in power maximization regime – optimal torque characteristic.
and the state space matrices are:

\[ A = \begin{bmatrix} \frac{1}{J} \cdot K_{\omega T_r} & -\frac{1}{J} \cdot n_{gb} \cdot a \\ \frac{n_{gb}}{T_e} & -\frac{1}{T_e} \end{bmatrix}, \quad (5.59) \]

\[ B_d = \begin{bmatrix} \frac{1}{J} \cdot K_{v T_r} \\ 0 \end{bmatrix}, \quad (5.60) \]

\[ C = \begin{bmatrix} n_{gb} \cdot \mu \cdot (a \Omega_{g0} + b) & a \cdot n_{gb} \cdot \mu \cdot \Omega_{r0} \\ K_{\omega F_T} & 0 \\ n_{gb}^2 \cdot \frac{J}{J} \cdot K_{\omega T_r} & n_{gb} \cdot \frac{J}{J} \cdot a \end{bmatrix}, \quad (5.61) \]

\[ D_b = \begin{bmatrix} 0 \\ K_{v F_T} \\ n_{gb}^2 \cdot \frac{J}{J} \cdot K_{v T_r} \end{bmatrix}, \quad (5.62) \]

The model was tested at operating point defined by \( P_{\text{dem}} = 5 \text{ MW} \) and \( v = 10.5 \text{ m/s} \). The results are given in Fig.5.24.

5.2.4. Power tracking below nominal generator speed

This operating regime emerges for a rather small set of operating conditions. It is characterized by saturated pitch angle:

\[ \beta = 0, \quad (5.63) \]

and the delivery of the demanded power:

\[ P = P_{\text{dem}}, \quad (5.64) \]

The relevant dynamics of this operating region is described by the state-space system where vectors are defined as:

\[ x = [\omega_r] \quad (5.65) \]

\[ u = [P_{\text{dem}}] \quad (5.66) \]

\[ d = [v] \quad (5.67) \]

\[ y = \begin{bmatrix} F_t \\ M_{\text{shaft}} \end{bmatrix}, \quad (5.68) \]
Figure 5.24: Simulation of the wind turbine in power maximization regime – affine torque characteristic.
and the state space matrices are:

\[
A = \left[ \frac{1}{J} \cdot K_\omega T_i + \frac{1}{J} \cdot n_{gb} \cdot \frac{P_{dem}}{\mu_{fg}} \right], \tag{5.70}
\]

\[
B = \left[ -n_{gb} \frac{1}{J} \cdot \frac{1}{\mu_{fg}} \right] \tag{5.71}
\]

\[
B_d = \left[ \frac{1}{J} K_v T_i \right]. \tag{5.72}
\]

\[
C = \left[ n_{gb}^2 \cdot \frac{1}{J} \cdot K_\omega T_i - n_{gb} \cdot \frac{P_{dem} n_{gb}}{\mu_{fg}} \right]. \tag{5.73}
\]

\[
D = \left[ 0 \frac{1}{\mu_{fg}} \right] \tag{5.74}
\]

\[
D_b = \left[ n_{gb}^2 \cdot \frac{1}{J} \cdot K_v T_i \right]. \tag{5.75}
\]

The model was tested at operating point defined by \( P_{dem} = 1.4 \text{ MW} \) and \( v = 7.5 \text{ m/s} \). The results are given in Fig. 5.25. The simulation was made for very small deviations from the operating point in order to keep the model in this operating regime, which is active for very small interval of inputs.

The figure shows that, unlike previous models, this one does not model the nonlinear system very good. Obviously, the change in power demand introduces a significant discrepancy. This discrepancy is the consequence of the constraints that are active in the original systems (and not modeled in the linearized model). Namely, when power demand is increased, the generator torque reference increases. This reduces the rotor speed (this is modeled by the linearized model). However, the linearized expression for aerodynamic torque that is used to derive the linearized model is not valid during this transient, because there is an implicit constraint on the maximal aerodynamic torque that is a function in rotational speed. This constraint is active until the rotational speed at a new static operating point is reached – the rotational speed at which the equation \( P_a(\omega \cdot R/v) = P_{dem} \) is valid. The same effect occurs due to changes in wind speed, but this is less apparent due to stochastic nature of the wind that tends to even out the discrepancy. This effect is indicated by much larger dynamic differences between the produced and demanded power.
Figure 5.25: Simulation of the wind turbine in power tracking mode below nominal generator speed.
Chapter 6

Conclusion

In this part of the Deliverable the wind turbine models provided for case studies are analyzed. The provided models are complex nonlinear hybrid models. A large number of subsystem contained in the V80 model are undisclosed and their functionality is unknown.

The aim of the presented work was to derive a model suitable for reconfigurable control design, that is a simple model that enables prediction of dynamic development of wind turbine production and loading at time scale of 1 s. The chosen approach was the identification of first principle model. Such model enables the generic approach to the problem and assures platform independency, to a certain extent.

Firstly, all wind turbine subsystems were modeled in detail. To obtain models of undisclosed blocks various system identification methods were applied. The wind turbine dynamic model that was obtained by integration of such sub-models proved to be an excellent approximation of provided model. The obtained complex model was then simplified in order to obtain model useful for reconfigurable control design. This resulted in linear models of very low order valid at operating points. Linear models were tested by simulation using various deterministic and turbulent winds. Results showed that very good trade-off between model simplicity and modeling error has been obtained.

While complex wind turbine model derived in this report offers excellent insight in wind turbine operation and control, derived linear models are the starting point for reconfigurable control design. Since a generic model is derived, it is possible to model the system in any operating point.

It is important to notice a somewhat unfortunate structure of the derived models, from the perspective of fatigue load minimization. Namely, both wind turbine models in power control operating mode (the most interesting one for wind farm control) have direct feed-through connection between the disturbance input – the wind speed – and the signals that contribute to fatigue load – the thrust force and the shaft moment. This is natural when wind turbine system is considered. The change in wind speed momentarily changes the thrust force and the rotor torque (which also momentarily changes the shaft moment). The control influence on thrust force can only come through change in pitch angle or change in rotor speed, which can not be reached directly by the control input. Therefore, the lack of
wind speed prediction model would probably lead to rather poor improvements in fatigue load based on thrust force because every change in wind speed would be visible as an oscillation in the thrust force. On the other hand, the shaft moment is also proportional to the generator torque (that is in power control regime directly influenced by the control input) so the shaft moment can be influenced by control action momentarily.
Part II

Reconfigurable control paradigm
Chapter 1

Introduction

This part of the report describes the control concept for wind farm power/load optimization with emphasis on compensation of disturbances present in a wind farm. In our wind farm control approach every wind turbine is treated as an individual power actuator affected by its own constraints. It is assumed that the demanded wind farm power production is given in advance, i.e., the power reference value for the whole farm is provided by the Transmission System Operator (TSO). The supervisory farm controller then distributes the power demand among wind turbines based on the dynamic model of wind field inside wind farm. The objective of distribution is to minimize the cumulative fatigue of all wind turbines in wind farm, while respecting the system constraints and fulfilling the TSO demands.

1.1. Supervisory wind farm control problem

The idea behind power/load optimization in Aeolus project is the employment of the wind field model that describes the dynamic development of wakes inside wind farm [5]. From a control perspective, the wakes impose coupling between individual wind turbines. Namely, a wind turbine standing in the wake created by an upwind turbine experiences reduced wind speed and increased turbulence, which affect its loading and power production potential. The motivation for the proposed wind farm power/load optimization is simple: the wake effects generated by any wind turbine can be changed by changing the turbine’s power reference, which itself is a manipulated (i.e., control) variable of a wind farm. By combining the wind field model with wind turbine model(s) we obtain the overall dynamic wind farm model suitable for optimization. Since this system involves many constraints, which are related mostly to wind turbine operation, and the goal is optimizing the production, a natural choice for the control design framework is the Model Predictive Control (MPC).

A centralized approach to the optimization of large wind farm operation is an extremely complex control problem. Namely, the governing system is best described as a coupled, constrained multiple-input multiple-output model whose order grows very fast with the size of wind farm. Furthermore, the wind turbine and especially the wind field are highly nonlinear systems. Also, the system is subjected to a large number of disturbances due to random
nature of wind, and diverse/random wind turbine malfunctions that may prevent or restrict its operation. Finally, wind farm model inherently comprises processes acting on very different time scales: the behavior of a typical megawatt scale wind turbine with local speed and power controller has dominant dynamics in the time scale of 1 second, while the typical propagation time of wind between two rows of wind turbines can be significantly longer than 10 seconds.

One of the primary demands on the wind farm controller set in the Aeolus project is the scalability of the control algorithms to wind farms of different sizes. We note that the wake effects significantly influence the wind farm behavior only in large wind farms (because in large wind farms most of the wind turbines are influenced by wakes), and for such farms computation of a centralized model predictive controller might be computationally intractable for practical, real-time applications. As an illustrative example consider the wind farm that comprises 16 rows of wind turbines separated by 200 meters. For wind speed of 15 m/s the prediction horizon should ideally be around 200 seconds in order to capture the wind propagation through the entire farm. With 1 second sampling time that would mean the solution to a huge (nonlinear) optimization problem – with prediction horizon of 200 steps one gets hundreds of optimization variables and constraints for every turbine in the farm – must be computed in less than 1 second. To circumvent the need to solve such complex optimization problems in this paper we propose the hierarchical supervisory control concept adapted to the application at hand.

1.2. Hierarchical wind farm control concept

The wind farm dynamics are effectively decoupled through different time scales, [6]. A two level hierarchical control concept is proposed that is utilizing this fact as a separation principle. The top level of control – the nominal supervisory controller – considers the propagation of mean wind stream through the wind farm and therefore accounts for the coupling that emerges due to wakes. This level of control derives the overall optimal wind farm operating point. The sampling time of this level can be adjusted to the size of wind farm and to separations of wind turbines inside a wind farm. The bottom level of control works at faster sampling rate and accommodates the nominal controller’s operating point to disturbances that occur on faster time scale, e.g. the influence of local wind gusts (i.e. the wind gusts that are not predicted by wind flow model) or sudden shut-down of a wind turbine. An implicit assumption is that the overall wind field in a wind farm and the short time scale disturbances are to a large extent decoupled. The intuitive reasoning is that because the local disturbances affect the wind field only locally then the operating point computed by the nominal controller is still valid for the largest part of the wind farm. Therefore, the objective of the disturbance compensation is to remain as close as possible to optimal operating point set by the nominal controller, while at the same time handling disturbances and respecting any newly imposed constraints.

The proposed control scheme is a two level hierarchical control concept with communication going only one way – top to bottom. The top level receives information (measurements) at slower sampling rates and the bottom level on faster sampling rates. The bottom level has a reduced computational effort since it does not need to handle the wind field model. Similar
control concept for multi time scale systems was described in [7].

This part of the report describes the hierarchical control concept for supervisory wind farm control. This control structure has been proposed in order to mitigate the computational load of the supervisory controller. Thanks to the time-scale decoupling one can achieve effective relaxation/decoupling of the optimal control problem. The design of the disturbance compensator resides on the following objectives:

- provide rejection of disturbances on fast time scale,
- respect system constraints,
- keep loads experienced by the wind turbine small, and
- remain as close as possible to distribution of power references provided by the nominal controller.

The idea of the solution that will be explained in more details in the following sections can be loosely summarized as follows.

1. For each wind turbine formulate a local finite time optimal control problem. Objective function has to penalize load experienced by wind turbine and deviation from power reference provided by the nominal control.

2. Obtain sensitivity functions of local costs and turbine loads to the changes in wind turbine power trajectory, where the real wind turbine state (measurements) are used as parameters.

3. Obtain parametric expression for the optimal controller. One group of parameters are the state measurements and the other define the power trajectory.

4. Impose global objective: \( \sum_{i=1}^{N} P_i = P_{TSO} \), where \( P_i \) is the power produced by the \( i \)-th wind turbine, \( P_{TSO} \) is the wind farm power demand provided by TSO and \( N \) is number of wind turbines in the wind farm.

5. Find such control actions that satisfy global objective and produce as small as possible increase in the local cost functions.

Note that computationally demanding steps 1–3 will be performed off-line.

This part of the report is structured as follows.

In Chapter 2 the theoretical background of the concepts that are used is provided. This includes theoretical background on mathematical programming, multi-parametric approach to mathematical programming and special classes of mathematical programm that will be used in the work.
In Chapter 3 the design of reconfigurable controller is described and the simulation results of the specific scenarios are provided and analyzed.
In Chapter 4 the work is concluded and the plan for the future work is provided.
Chapter 2

Constrained Finite Time Optimal Control – multi-parametric solution

This chapter briefly summarizes the theoretical background that are used in the work. The concept of multi-parametric solution of mathematical program is introduced and related to the problem of Constrained Finite Time Optimal Control (CFTOC). Some typical formulations of multi-parametric programs are introduced.

2.1. Basic definitions

In this section the basic terminology and concepts are defined.

Definition 2.1 (Mathematical Program). \[8,9\] A mathematical program is an optimization problem of the form

\[
\begin{align*}
\inf_z \ & f(z) \\
\text{subj. to} \ & \begin{cases} 
g(z) \leq 0, \\
h(z) = 0, \\
z \in Z,
\end{cases}
\end{align*}
\]

where \( z \in \mathbb{R}^{n_z} \) is the optimization variable, while the functions \( f : \mathbb{R}^{n_z} \to \mathbb{R} \), \( g : \mathbb{R}^{n_z} \to \mathbb{R}^{n_g} \), and \( h : \mathbb{R}^{n_z} \to \mathbb{R}^{n_h} \), and the set \( Z \subseteq \mathbb{R}^{n_z} \) are given problem parameters.

The function \( f \) is called the cost function or objective function. The relations \( g(z) \leq 0 \), \( h(z) = 0 \), and \( z \in Z \) are called (resp. inequality, equality and set) constraints. The set of all points for which real valued functions \( f \), \( g \) and \( h \) are defined is called the domain of the problem (2.2), i.e. domain of (2.2) = \( \text{dom}(f) \cap \text{dom}(g) \cap \text{dom}(h) \). A point \( z \) in the domain is feasible if it satisfies all constraints. The set of all feasible points is called the feasible set. We say that the problem (2.2) is feasible if there is at least one feasible point, and infeasible if there are no feasible points. A point \( z^* \) is optimal if it is feasible and if \( f(z^*) \leq f(z) \) for all feasible \( z \). If there are no constraints \( n_g = 0, n_h = 0 \) and \( Z = \mathbb{R}^{n_z} \) we say that the problem
(2.2) is unconstrained. If there are feasible points $z_k$ with $f(z_k) \to -\infty$ as $k \to \infty$ we say that the problem (2.2) is unbounded (below).

The mathematical program is a very general problem description. Next, a special class of mathematical program will be defined.

**Definition 2.2** (Convex Program). A mathematical program is called the *convex program* if it has the form

$$\min_z f(z) \quad \text{(CP)}$$

subj. to \[
\begin{align*}
g(z) &\leq 0, \\
z &\in \mathcal{Z},
\end{align*}
\]

where $z \in \mathbb{R}^{n_z}$ is the optimization variable, the set $\mathcal{Z} \subseteq \mathbb{R}^{n_z}$ is convex, the cost function $f : \mathbb{R}^{n_z} \to \mathbb{R}$ is convex, and all components of the constraint function $g : \mathbb{R}^{n_z} \to \mathbb{R}^{n_g}$ are convex functions.

For definitions of convex functions and convex sets see [10]. The convex program is a special class of mathematical program that has a well established theory and there are relatively efficient algorithms for solving a general convex optimization problems. Further we define some specific instances of convex programs. Very efficient algorithms were developed for solving such program formulations.

**Definition 2.3** (Linear Program). A *linear program* is a convex optimization problem that can be expressed in the form

$$\min_z c'z \quad \text{(LP)}$$

subj. to \[
G^z z \leq G,
\]

where $z \in \mathbb{R}^{n_z}$ is the optimization variable, and matrices $c \in \mathbb{R}^{n_z}$, $G^z \in \mathbb{R}^{n_g \times n_z}$, $G \in \mathbb{R}^{n_G}$ are given problem parameters.

Theoretically every LP (with rational parameters) is solvable in polynomial time by both the ellipsoid method of Khachiyan [11, 12] and various interior point methods [13, 14]. A practical algorithm to solve an LP with $n$ variables and $m$ constraints requires roughly $O(n^3 m^{0.5} + n^2 m^{1.5})$ operations [15].

**Definition 2.4** (Quadratic Program). A *quadratic program* is the convex optimization problem that can be expressed in the form

$$\min_z 0.5z'Qz + c'z \quad \text{(QP)}$$

subj. to \[
G^z z \leq G,
\]

where $z \in \mathbb{R}^{n_z}$ is the optimization variable, while $Q \in \mathbb{R}^{n_z \times n_z}$, $Q = Q' \succeq 0$, $c \in \mathbb{R}^{n_z}$, $G^z \in \mathbb{R}^{n_G \times n_z}$, $G \in \mathbb{R}^{n_G}$.
QPs can be solved with roughly the same efficiency as LPs, i.e. the solvers are approximately 5-times slower than an LP solver [16, page 37].

When some of the optimization variables in a linear (quadratic) program are constrained to integer values the ensuing problem is called a mixed integer linear (quadratic) program.

Definition 2.5 (Mixed Integer Linear Program). A *mixed integer linear program* (MILP) is a non-convex optimization problem that can be expressed in the form
\[
\min_z \ f'z \\
\text{subj. to} \ Gz \leq G,
\]
where \(z \in \mathbb{R}^{n_r} \times \{0, 1\}^{n_b}\) is the optimization variable, \(n_r\) is the number of real valued variables, \(n_b\) is the number of binary (or, in general, integer) variables, and matrices \(f \in \mathbb{R}^{n}, G \in \mathbb{R}^{n_G \times n}\), with \(n = n_r + n_b\), are given problem parameters.

Definition 2.6 (Mixed Integer Quadratic Program). A *mixed integer quadratic program* (MIQP) is a non-convex optimization problem that can be expressed in the form
\[
\min_z \ 0.5z'Qz + c'z \\
\text{subj. to} \ Gz \leq G,
\]
where \(z \in \mathbb{R}^{n_r} \times \{0, 1\}^{n_b}\) is the optimization variable, \(n_r\) is the number of real valued variables, \(n_b\) is the number of binary (or, in general, integer) variables, and matrices \(f \in \mathbb{R}^{n}, G \in \mathbb{R}^{n_G \times n}, G \in \mathbb{R}^{n_G \times n}, c \in \mathbb{R}^{n_c}\), with \(n = n_r + n_b\), are given problem parameters.

In the following we define the terminology that will be used to characterize solutions of the analyzed mathematical programs.

Definition 2.7 (Polyhedron). [17] A convex set \(S \subseteq \mathbb{R}^n\) given as an intersection of a finite number of closed half-spaces
\[
S = \{x \in \mathbb{R}^n \mid S^x x \leq S^c\}, \tag{2.3}
\]
is called *polyhedron*. Here, the inequality \(S^x x \leq S^c\), with \(S^x \in \mathbb{R}^{n_S \times n}, S^c \in \mathbb{R}^n, n_S < \infty\), is considered component-wise.

Definition 2.8 (Polytope). [17] A bounded polyhedron \(P \subset \mathbb{R}^n\)
\[
P = \{x \in \mathbb{R}^n \mid P^x x \leq P^c\}, \tag{2.4}
\]
is called *polytope*. Here, the inequality \(P^x x \leq P^c\), with \(P^x \in \mathbb{R}^{n_P \times n}, P^c \in \mathbb{R}^{n_P}, n_P < \infty\), is considered component-wise.

Definition 2.9. A function \(h : \Theta \to \mathbb{R}^k\), where \(\Theta \subseteq \mathbb{R}^s\), is *piecewise affine (PWA)* if there exists a partition \(R_1, \ldots, R_N\) of \(\Theta\) and \(h(\theta) = H^i\theta + h^i, \forall \theta \in R_i, i = 1, \ldots, N\).

Definition 2.10. A function \(h : \Theta \to \mathbb{R}^k\), where \(\Theta \subseteq \mathbb{R}^s\), is *PWA on polyhedra (PPWA)* if there exists a polyhedral partition \(R_1, \ldots, R_N\) of \(\Theta\) and \(h(\theta) = H^i\theta + h^i, \forall \theta \in R_i, i = 1, \ldots, N\).

Piecewise quadratic functions (PWQ) and piecewise quadratic functions on polyhedra (PPWQ) are defined analogously.
2.2. Multi-parametric programming

Consider the nonlinear mathematical program dependent on a parameter vector $x$ appearing in the cost function and in the constraints

$$J^*(x) = \inf_{z} f(z, x)$$
subject to
$$g(z, x) \leq 0$$
$$z \in M,$$

where $z \in \mathbb{R}^s$ is the optimization vector, $x \in \mathbb{R}^n$ is the parameter vector, $f : \mathbb{R}^s \times \mathbb{R}^n \to \mathbb{R}$ is the cost function, $g : \mathbb{R}^s \times \mathbb{R}^n \to \mathbb{R}^n$ are the constraints and $M \subseteq \mathbb{R}^s$.

A small perturbation of the parameter $x$ in (2.5) can cause a variety of outcomes, i.e., depending on the properties of the functions $f$ and $g$ the solution $z^*(x)$ may vary smoothly or change abruptly as a function of $x$. We denote by $K^*$ the set of feasible parameters, i.e.,

$$K^* = \{ x \in \mathbb{R}^n | \exists z \in M, g(z, x) \leq 0 \},$$

by $R : \mathbb{R}^n \to 2^{\mathbb{R}^s}$, where $2^{\mathbb{R}^s}$ denotes the set of all subsets of $\mathbb{R}^s$, the point-to-set map that assigns the set of feasible $z$

$$R(x) = \{ z \in M | g(z, x) \leq 0 \}$$

and by $Z^* : K^* \to 2^{\mathbb{R}^s}$, the point-to-set map which expresses the dependence on $x$ of the set of optimizers, i.e., $Z^*(x) = \{ z \in R(x) | f(z, x) = J^*(x) \}$ with $\bar{x} \in K^*$.

$J^*(x)$ will be referred to as the optimal value function or simply value function, $Z^*(x)$ will be referred to as the optimal set. We will denote by $z^* : \mathbb{R}^n \to \mathbb{R}^s$ one of the possible single valued functions that can be extracted from $Z^*$, $z^*$ will be called the optimizer function. If $Z^*(x)$ is a singleton for all $x$, then $z^*(x)$ is the only element of $Z^*(x)$. Throughout the paper $z^*$ is a real vector-valued function, although more generally we will assume that infinite values may be allowed.

Fiacco ([18, Chapter 2]) provides conditions under which the solution of nonlinear multi-parametric programs (2.5) is locally well behaved and establishes properties of the solution as a function of the parameters. In the following we report a basic result [19] which focuses on a restricted set of functions $f(z, x)$ and $g(z, x)$:

**Theorem 2.1** ([19]). Consider the multi-parametric nonlinear program (2.5). Assume that $M$ is a convex and bounded set in $\mathbb{R}^s$, $f$ is continuous and the components of $g$ are convex on $M \times \mathbb{R}^n$. Then, $J^*(x)$ is continuous at each $x \in \text{relint}(K^*)$. 

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Theorem 2.2. Consider the multi-parametric nonlinear program (2.5). Assume that $M$ is a convex and bounded set in $\mathbb{R}^s$ and that $f$ is continuous and the components of $g$ are quasi-convex on $M \times \mathbb{R}^n$. Then, $J^*(x)$ is continuous at each $x$ belonging to the relative interior of $K^*$.

Theorem 2.3 ([18]). Consider the multi-parametric nonlinear program (2.5). If in addition to the assumptions of Theorem 2.1 we assume that $f$ is strictly quasi-convex in $z$ for each fixed $x$, then $z^*$ is a continuous real-valued function.

Further we restrict our attention to two special classes of multi-parametric programming.

2.2.1. Multi-parametric Linear Program

Consider the multi-parametric program

$$J^*(x) = \min_z g'z \quad \text{s.t.} \quad Cz \leq c + Sx,$$

where $z \in \mathbb{R}^{n_z}$ is the optimization vector, $x \in \mathbb{R}^n$ is the vector of parameters, and $g \in \mathbb{R}^{n_z}$, $C \in \mathbb{R}^{q \times n_z}$, $c \in \mathbb{R}^q$, $S \in \mathbb{R}^{q \times n}$ are constant matrices. We refer to (2.9) as a (right-hand-side) multi-parametric linear program (mp-LP) [22,23].

Theorem 2.4 ([22]). Consider the mp-LP (2.9). The set $K^*$ is a polyhedral set, the value function $J^* : K^* \to \mathbb{R}$ is PPWA, convex and continuous and there exists a continuous and PPWA optimizer function $z^* : K^* \to \mathbb{R}^{n_z}$.

Proof See [22]. □

When in (2.9) we add constraints that restrict some of the optimization variables to be 0 or 1, $z := [z_c, z_\ell]$, where $z_c \in \mathbb{R}^{n_c}$, $z_\ell \in \{0,1\}^{n_\ell}$, we refer to (2.9) as a (right-hand-side) multi-parametric mixed-integer linear program (mp-MILP) [24].

Theorem 2.5. Consider the mp-MILP (2.9). The set $K^*$ is a (possibly non-convex and disconnected) polyhedral set, the value function $J^* : K^* \to \mathbb{R}$ is PPWA and there exist PPWA optimizer functions $z_c^* : K^* \to \mathbb{R}^{n_c}$, $z_\ell^* : K^* \to \{0,1\}^{n_\ell}$

Proof Easily follows from the Algorithm described in [24]. □

2.2.2. Multi-parametric Quadratic Program

Consider the multi-parametric program

$$J^*(x) = \frac{1}{2}x'Yx + \min_z \frac{1}{2}z'Hz + z'Fx \quad \text{subj. to} \quad Cz \leq c + Sx,$$

where $Y, H, F \in \mathbb{R}^{n \times n}$ are constant matrices.

We define here a non-convex polyhedral set as a non-convex set given by the union of a finite number of convex polyhedra with mutually disjoint interiors.
where \( z \in \mathbb{R}^{n_z} \) is the optimization vector, \( x \in \mathbb{R}^n \) is the vector of parameters, and \( C \in \mathbb{R}^{q \times n_z} \), \( c \in \mathbb{R}^q, S \in \mathbb{R}^{q \times n} \) are constant matrices. We refer to the problem of computing \( z^*(x) \) and \( J^*(x) \) in (2.10) as (right-hand-side) multi-parametric quadratic program (mp-QP).

**Theorem 2.6** ([25]). Consider the mp-QP (2.10). Assume \( H \succ 0 \) and \( \begin{bmatrix} Y & F' \end{bmatrix} F H \succeq 0 \). The set \( K^* \) is a polyhedral set, the value function \( J^* : K^* \to \mathbb{R} \) is PPWQ, convex and continuous and the optimizer \( z^* : K^* \to \mathbb{R}^{n_z} \) is PPWA and continuous.

When in (2.10) we add constraints that restrict some of the optimization variables to be 0 or 1, \( z := [z_c, z_\ell] \), where \( z_c \in \mathbb{R}^{n_c} \), \( z_\ell \in \{0, 1\}^{n_\ell} \), we refer to a (right-hand-side) multi-parametric mixed-integer quadratic program (mp-MIQP).

**Theorem 2.7** ([26]). Consider the mp-MIQP (2.10). The set \( K^* \) is a (possibly non-convex and disconnected) polyhedral set, the value function \( J^* : K^* \to \mathbb{R} \) is PWQ and there exist PWA optimizer functions \( z^*_c : K^* \to \mathbb{R}^{n_c} \), \( z^*_\ell : K^* \to \{0, 1\}^{n_\ell} \).

### 2.3. Constrained Finite Time Optimal Control of Linear Systems

#### 2.3.1. CFTOC problem formulation

Consider the discrete-time linear time-invariant system

\[
x(t + 1) = Ax(t) + Bu(t)
\]

subject to the constraints

\[
E^x x(t) + E^u u(t) \leq E
\]

at all time instants \( t \geq 0 \).

In (2.11)–(2.12), \( n_x \in \mathbb{N} \), \( n_u \in \mathbb{N} \) and \( n_E \in \mathbb{N} \) are the number of states, inputs and constraints respectively, \( x(t) \in \mathbb{R}^{n_x} \) is the state vector, \( u(t) \in \mathbb{R}^{n_u} \) is the input vector, \( A \in \mathbb{R}^{n_x \times n_x} \), \( B \in \mathbb{R}^{n_x \times n_u} \), \( E^x \in \mathbb{R}^{n_E \times n_x} \), \( E^u \in \mathbb{R}^{n_E \times n_u} \), \( E \in \mathbb{R}^{n_E} \), the pair \( (A, B) \) is stabilizable, and the vector inequality (2.12) is considered elementwise.

Let \( x_0 = x(0) \) be the initial state and consider the constrained finite-time optimal control problem

\[
J^*(x_0) := \min_U J(x_0, U)
\]

subject to

\[
\begin{align*}
x_{k+1} &= Ax_k + Bu_k, \quad k \geq 0, \\
E^x x_k + E^u u_k &\leq E, \quad k = 0, \ldots, N - 1,
\end{align*}
\]

where \( N \in \mathbb{N} \) is the horizon length, \( U := [u'_0, \ldots, u'_{N-1}]' \in \mathbb{R}^{n_u N} \) is the optimization vector, \( x_i \) denotes the state at time \( i \) if the initial state is \( x_0 \) and the control sequence \( \{u_0, \ldots, u_{N-1}\} \) is applied to the system (2.11), \( J^* : \mathbb{R}^{n_z} \to \mathbb{R} \) is the value function, and the cost function
$J : \mathbb{R}^{n_x} \times \mathbb{R}^{n_uN} \rightarrow \mathbb{R}$ is given either as a polyhedral linear function (i.e., sum of linear norms)

$$J(x_0, U) = \|Q^{xN} x_N\|_\ell + \sum_{k=0}^{N-1} \|Q^x x_k\|_\ell + \|Q^u u_k\|_\ell, \quad \ell \in \{1, \infty\},$$

(2.14a)

or as a quadratic function

$$J(x_0, U) = x_N' Q^{xN} x_N + \sum_{k=0}^{N-1} x_k' Q^x x_k + u_k' Q^u u_k.$$  

(2.14b)

In the following, we will assume that $Q^x$, $Q^u$, $Q^{xN}$ are full column rank matrices when the cost function (2.14a) is used, and that $Q^x = (Q^x)' \succeq 0$, $Q^u = (Q^u)' \succ 0$, $Q^{xN} \succeq 0$, when the cost function (2.14b) is used.

The optimization problem (2.13) can be translated into a linear program (LP) when the linear cost function (2.14a) is used [27] or into a quadratic program (QP) when the quadratic cost function (2.14b) is used [25]. The optimizer $U^* = [(u^*_0)' \ldots (u^*_{N-1})]'$ of problem (2.13)–(2.14) is a function of the initial state $x_0$. It can be computed by solving an LP or a QP once $x_0$ is fixed or it can be computed explicitly for all $x_0$ within a given range of values as explained in the following.

### 2.3.2. Receding Horizon Control strategy

Consider the problem of regulating to the origin the discrete-time linear time-invariant system (2.11) while fulfilling the constraints (2.12). The solution $U^*$ to CFTOC problem (2.13)–(2.14) is an open-loop optimal control trajectory over a finite horizon. A receding horizon control strategy employs it to obtain a feedback control law in the following way: Assume that a full measurement of the state $x(t)$ is available at the current time $t \geq 0$. Then, the CFTOC problem (2.13)–(2.14) is solved at each time $t$ for $x_0 = x(t)$, and

$$u(t) = u^*_0$$

(2.15)

is applied as an input to system (2.11).

The two main issues regarding this policy are the feasibility of the optimization problem (2.13)–(2.14) for all $t \geq 0$ and the stability of the resulting closed-loop system. We will assume that the matrices $Q^x$, $Q^u$, $Q^{xN}$, the horizon length $N$ and the constraints in (2.13) have been chosen to guarantee the stability and the feasibility of RHC control law (2.13)–(2.15). For a detailed discussion see, e.g., [25, 27–31].

### 2.3.3. Solution of CFTOC, linear cost Case

Consider the problem (2.13) with the linear cost function (2.14a) and $\ell = \infty$. Using a standard transformation [27], introducing the vector $v := [u^*_0, \ldots, u^*_{N-1}, \varepsilon^x_1, \ldots, \varepsilon^x_N, \varepsilon^u_1, \ldots, \varepsilon^u_N]' \in \mathbb{R}^{n_v}$,
The RHC

2.3.4. Solution of CFTOC, quadratic cost Case

∀ = 1, i

available explicitly, as the optimal input from Q as a piecewise affine function of two different ways: solve the LP (2.16) on-line at each time step for a given where

Corollary 2.1.

Theorem 2.8.

Once the multi-parametric problem (2.16) has been solved off-line for a polyhedral set R of states, the explicit solution of states, the explicit solution is available within a given range of values, i.e., by considering (2.16) as a multi-parametric linear program (mp-LP) [22].

Solving an mp-LP means computing the optimizer v*(x) and the value function J*(x) for all possible vectors x in a given set X. The solution to mp-LP problems can be simply approached by exploiting the properties of the primal and dual optimality conditions as proposed in [22,23].

Once the multi-parametric problem (2.16) has been solved off-line for a polyhedral set X ⊆ Rn, the explicit solution v*(x) of CFTOC problem (2.16) is available as a piecewise affine function of x, and the receding horizon controller (2.13)–(2.15) is also available explicitly, as the optimal input u(t) consists simply of na components of v*(x(t))

u(t) = [I na 0 . . . 0]v*(x(t)).

(2.17)

In [22] the following results about the properties of the solution are proved:

Theorem 2.8. Consider the multi-parametric linear program (2.16). Then the set of feasible parameters Xf is convex. If the optimizer v*(x) is unique for all x ∈ Xf, then the optimizer function v* : Xf → Rna is continuous and piecewise affine. Otherwise, it is always possible to define a continuous and piecewise affine optimizer function v*(x) for all x ∈ Xf.

Corollary 2.1. The RHC (2.17), defined by the optimization problem (2.13), (2.14a) and (2.15), is a continuous and piecewise affine function, u : Rnx → Rna, and has the form

u(x) = Fi x + Gi, ∀x ∈ Pi, i = 1, . . . , NP,

where Fi ∈ Rna×nx, Gi ∈ Rna, and Pi = {x ∈ Rnx | Pi x ≤ Pi}, Pi ∈ Rna×nx, Pi ∈ Rna, i = 1, . . . , NP is a polyhedral partition of Xf (∪i=1 NP Pi = Xf, and Pi, Pj have disjoint interiors ∀i ̸= j).

2.3.4. Solution of CFTOC, quadratic cost Case

Consider the problem (2.13) with the quadratic cost function (2.14b). By substituting x_k = A^k x_0 + \sum_{j=0}^{k-1} A^j B u_{k-1-j} in (2.13)–(2.14), this can be rewritten as the quadratic program

\[ J^*(x) = \frac{1}{2} x' Y x + \min_{U} \frac{1}{2} U' H U + x' F U \]

subj. to \[ M U \leq M + M' x \]

The same holds for l = 1 with a different optimizer vector [27].
where \( x = x_0 \), the column vector \( U := [u'_0, \ldots, u'_{N-1}]' \in \mathbb{R}^{nu} \), \( n_U := n_uN \), is the optimization vector, \( H = H' \succ 0 \), and \( H, F, Y, M^U, M^x, M \) are easily obtained from \( Q^x, Q^u, Q^{xN} \) and (2.13)–(2.14) (see [25] for details).

As in the linear cost case, because the problem depends on \( x \) the implementation of RHC can be performed either by solving the QP (2.19) on-line or, as shown in [25,32], by solving problem (2.19) off-line for all \( x \) within a given range of values, i.e., by considering (2.19) as a multi-parametric Quadratic Program (mp-QP).

Once the multi-parametric problem (2.19) is solved off-line, i.e., the solution \( U^*(x) \) of the CFTOC problem (2.19) is found, the state-feedback PWA RHC law is simply
\[
u(t) = [I_{nu} 0 \ldots 0]U^*(x(t)).
\]

The proof of the following results about the properties of the solution is provided in [25].

**Theorem 2.9.** Consider the multi-parametric quadratic program (2.19) and let \( H \succ 0 \). Then the set of feasible parameters \( \mathcal{X}_f \) is convex, the optimizer \( U^* : \mathcal{X}_f \to \mathbb{R}^s \) is continuous and piecewise affine, and the optimal solution \( J^* : \mathcal{X}_f \to \mathbb{R} \) is continuous, convex and piecewise quadratic.

**Corollary 2.2.** The RHC control law (2.20), defined by the optimization problem (2.13), (2.14b) and (2.15), is continuous and piecewise affine, and has the form (2.18).

Corollaries 2.1 and 2.2 state that by using a multi-parametric solver the computation of RHC action becomes a simple piecewise affine function evaluation.
Chapter 3

Wind farm control problem

The schematics of the hierarchical supervisory control system that is pursued in Work Package 3 is presented in Fig. 3.1. This conceptual set-up is taken over from [7], where it is presented as an architecture for hierarchical Model Predictive Control systems with separable slow and fast dynamics.

The fast dynamics that influences wind farm control was analyzed in [33]. The eminent issues that fast scale wind farm controller needs to handle are the wind turbine shut-down and occurrence of wind gusts. In the case of a wind turbine shut-down the task of the fast scale wind farm controller is to redistribute the power references in order to satisfy the global objective of the wind farm control and that is to deliver the required amount of power from the wind farm with minimal possible overall fatigue load. One might say that this scenario requires rearrangement or reconfiguration of power reference distribution (this is the origin of the term reconfigurable control extension).

As it was shown in the first part of this Deliverable, the occurrence of a positive wind gusts (for the type of wind turbine and wind turbine control system that are provided as case study models) does not influence the power production of a wind turbine (if a wind turbine is in a power-controlled mode). Nevertheless, the wind gust causes oscillation in wind turbine moments that define its fatigue. Therefore, fast controller can try to improve performance during wind gusts, to alleviate the disturbance.

The Fig. 3.2 depicts the conceptual design of the reconfigurable controller.

![Figure 3.1: Control concept for systems with slow and fast dynamics [7].](image)

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The black lines on the figure present the computations and designs done off-line. Those are:

1. **Determine local objective** – the cost function that models desired behavior of a single wind turbine. Based on the cost function the vector of parameters is constructed.

2. **Pose a CFTOC problem for a single wind turbine as a multi-parametric program.** The parameters include: system measurements, measured disturbances (the wind speed) and the wind turbine reference.

3. **Compute multi-parametric solution of the CFTOC – a polyhedral set \( \Theta \), the value function \( J^* : \Theta \to \mathbb{R} \) and the optimizer \( z^* : \Theta \to \mathbb{R}^{n_z} \).

4. **Design the algorithm for reference generation** – the algorithm that generates the references for reconfigurable control problem based on references provided by supervisory controller.

5. **Design the reconfiguration algorithm** – the algorithm that will, based on the sensitivity of computed value functions on the wind turbine reference, determine the wind farm control signals.

The blue lines carry the wind farm references from supervisory control. These are updated at the sampling rate that is used by supervisory controller.

The red lines show the computations that are done at every sampling instant of the reconfigurable control. Those are:

1. **Collect system measurements.** Those measurements are a part of the parameter of multi-parametric CFTOC solution.

2. **Fix the following parameters: the state measurements, the disturbance measurements and state and output references** and thus obtain the \( n_r \) dimensional polyhedral set, and the associated optimizer and value function, where \( n_r \) is the dimension of the wind turbine control reference \( (u_{\text{ref}}) \).

3. **Reconfiguration algorithm.** The references are redistributed based on the obtained value functions and the optimal control input is sent to the wind turbines.

### 3.1. Local control formulation

The task of the inner control loop in the control concept on Fig. 3.1 is to track the references provided by the outer control loop. Therefore, the local control objective is constructed to penalize the offset of the outputs, states and inputs from their reference value in the prediction horizon. This general control problem can be cast as a CFTOC problem:

\[
\begin{align*}
\min_U \quad & J(U) = (Y - Y_{\text{ref}})'Q_y(Y - Y_{\text{ref}}) + (X - X_{\text{ref}})'Q_x(X - X_{\text{ref}}) + (U - U_{\text{ref}})'R(U - U_{\text{ref}}) \\
\text{subj. to} \quad & \begin{cases} 
X = Ax_0 + BU + B_dD + F, \\
Y = Cx_0 + DU + D_dD + G, \\
E^U U + E^x x_0 \leq E,
\end{cases}
\end{align*}
\]

(3.1)
Figure 3.2: Design of the reconfigurable controller.
where:

\[ Y = \begin{bmatrix} y_1' & y_2' & \ldots & y_{N-1}' \end{bmatrix}' \]

is the vector of predicted outputs,

\[ X = \begin{bmatrix} x_1' & x_2' & \ldots & x_{N-1}' \end{bmatrix}' \]

is the vector of predicted states,

\[ U = \begin{bmatrix} u_1' & u_2' & \ldots & u_{N-1}' \end{bmatrix}' \]

is the vector of predicted inputs,

\[ Y_{\text{ref}} = \begin{bmatrix} y_{\text{ref},1}' & y_{\text{ref},2}' & \ldots & y_{\text{ref},N-1}' \end{bmatrix}' \]

is the vector of output references,

\[ X_{\text{ref}} = \begin{bmatrix} x_{\text{ref},1}' & x_{\text{ref},2}' & \ldots & x_{\text{ref},N-1}' \end{bmatrix}' \]

is the vector of state references,

\[ U_{\text{ref}} = \begin{bmatrix} u_{\text{ref},1}' & u_{\text{ref},2}' & \ldots & u_{\text{ref},N-1}' \end{bmatrix}' \]

is the vector of input references,

\[ Q_y, Q_x, R \text{ are the weight matrices, } Q_y' \geq 0, Q_x^x = (Q_x')' \geq 0, Q_u^u = (Q_u')' \succ 0, \]

\[ Q_{xN} \succeq 0, \]

\[ x_0 \text{ is the initial system state}, \]

\[ D = \begin{bmatrix} d_0' & d_1' & d_2' & \ldots & d_{N-1}' \end{bmatrix}' \]

is the vector of predicted disturbances, and

\[ E^U, E^x, E \text{ define system constraints.} \]

The equality constraints in (3.1) are the system prediction model, obtained from the wind turbine model linearized at one of the chosen operating point.

In (3.1) the affine prediction model is used instead of the linear model. This form of the model allows the description of the model around the real operating point instead of transferring the model to the origin of the state space. The use of such model instead of the linear does not present a difficulty in casting a CFTOC as a quadratic program. Computationally, it is an equivalent action to translating the references to the origin, but this form of the state description allows more natural description of the problem.

In this report we use persistent wind model for wind speed prediction:

\[ v[k+1] = v[k] = v, \quad (3.2) \]

where \( v \) denotes the measured (or estimated) wind speed, and we use no preview of references:

\[ y_{\text{ref}}[k+1] = y_{\text{ref}}[k] = y_{\text{ref}}, \quad x_{\text{ref}}[k+1] = x_{\text{ref}}[k] = x_{\text{ref}}, \quad u_{\text{ref}}[k+1] = u_{\text{ref}}[k] = u_{\text{ref}}. \quad (3.3) \]

Also, at this point we include only the constraints on the control signal:

\[ U_{\text{min}} \leq U \leq U_{\text{max}}, \quad (3.4) \]

where:

\[ U_{\text{min}} = \begin{bmatrix} u_{\text{min}} & \ldots & u_{\text{min}} \end{bmatrix}' \]

is the vector that contains minimal power that can be produced by a wind turbine in the prediction horizon (a constant value that depends on the wind turbine construction), and
$U_{\text{max}} = [u_{\text{max}} \cdots u_{\text{max}}]'$ is the vector that contains maximal available power that can be produced by a wind turbine in the prediction horizon (since we use persistent wind model $u_{\text{max}}$ is constant for the prediction horizon).

The purpose of the local controller defined in this way as a self standing controller (i.e. as not a part of the wind farm controller) is not very natural. This choice of control design gains its real purpose as a trade-off mechanism for distribution of references inside a wind farm. However, to depict its functionality, the system’s response for fictional references is depicted on Fig. 3.3 is provided. It shows the response of a locally controlled wind turbine in a case of a wind disturbance in comparison to the system to which the $u_{\text{ref}}$ was applied as a control signal. The disturbance is a stepwise increase followed by stepwise decrease in wind speed (a simple approximation of a wind gust). It can be observed that during the nominal conditions local controller has no influence on wind turbine behavior. However, as the wind changes the outputs of the model (thrust force and shaft moment) change. The local controller changes the control variable (moves it away from $U_{\text{ref}}$). By the tuning of the weights the relative importance can be given to offset of different sizes. In the example shown in Fig. 3.3 the largest weight was put on thrust force tracking. It can be seen that very good tracking of thrust force was obtained by adapting the control input $P_{\text{dem}}$. A very good effect from the point of minimizing fatigue is that this control concept reduces the oscillatory behavior of the penalized outputs. Unfortunately, the wind turbine mechanics as well as the design of the local power control system are such that during wind gust a large leap in the thrust force can not be avoided. This is obvious from the state-space models derived in Part I of the report, where wind speed has a direct feed-through to thrust force. Also, increase in the control variable causes a proportional jump in the generator torque.

For use in the overall wind farm control the local optimization problem (3.1) can be restated as an mpQP program:

$$J^*(\theta) = \min_{\theta} \quad J(\theta, U)$$
$$\text{subj. to} \quad E^U U + E^\theta \theta \leq E,$$

(3.5)

where $\theta = [x_0' \quad v' \quad u_{\text{ref}}' \quad x_{\text{ref}}' \quad y_{\text{ref}}' \quad u_{\text{max}}']'$ is the parameter vector of the problem. The parameters $\theta$ are the signals that change in time during reconfigurable control operation. The state vector $x_0$ and the wind speed (disturbance) $v$ will be measured at every time instant, the references $u_{\text{ref}}$, $x_{\text{ref}}$ and $y_{\text{ref}}$ are provided from the supervisory controller, and $u_{\text{max}}$ is computed at every time instant as a function of the measurements $x_0$ and $v$ according to the expressions for the maximal wind turbine power derived in the first part of the report.

We note here that if the supervisory control algorithm is designed in a way that it can only provide the reference for the control signal (and not for the state and output), or a model used by the supervisory controller lacks precision due to its simplicity, the algorithm described in Part I. Alg. 4.1 will be used to determine the state and output references based on the control signal provided by supervisory controller and the measurements that the supervisory controller used for optimal control computation.

It can be argued that the maximal power can be cast as a function of other parameters in the parameter vector and that it is therefore redundant. Since it was shown in the first
Figure 3.3: Local controller behavior during disturbances.
part of the deliverable that the maximal wind turbine power is a nonlinear function that
has two very different regions (nonlinear function for \( P_{\text{max}} < P_{\text{dem}} \) and \( P_{\text{max}} = P_{\text{dem}} \))
this function can not be approximated by a linear function. Since inclusion of PWA constraint
would lead to MIQP formulation of the problem that we are avoiding for the moment, the
maximal power is introduced as a separate parameter and its value will be computed at every
time instant by using the algorithm Part I. Alg. 4.1.

The optimization problem (3.5) is a multi-parametric quadratic program and can be
easily cast into the form (2.10). It is solved off-line to obtain the polyhedral set \( \Theta \), the value
function \( J^* : \Theta \rightarrow \mathbb{R} \), and the optimizer \( U^* : \Theta \rightarrow \mathbb{R}^{n_z} \). According to Th. 2.6, the optimizer
is a continuous PPWA function, and the value function is a convex and continuous PPWQ
function. We use this characteristics of the value function and the optimizer to define the
reconfiguration algorithm.

3.2. Reconfiguration algorithm

The wind farm model used by the reconfiguration control neglects interconnections between
wind turbines, i.e., the wind farm is treated as a set of individual wind turbines. Let \( U^{*j}(\cdot) \)
and \( J^{*j}(\cdot) \) be the optimizer and the value function expressions for the \( j \)-th wind turbine,
and let \( \Theta^j \) be the space of parameters \( (\theta = [x^j_0 \ v^j \ u^j_{\text{ref}} \ x^j_{\text{ref}} \ y^j_{\text{ref}} \ u^j_{\text{max}}]) \) for which those
expressions are valid. Note that we use superscript \( j \) to denote variables related to the \( j \)-th
wind turbine, see Fig. 3.2.

At the time \( t \), the state vector \( x^j_0(t) \), the wind speed (disturbance) \( v^j(t) \), the references
\( x^j_{\text{ref}}(t) \) and \( y^j_{\text{ref}}(t) \) (provided by the supervisory controller), and \( u^j_{\text{max}}(t) \), are known and fixed.
Then (from a pre-solved local control problem (3.5)) we obtain the optimizer

\[
    z^j(U^j_{\text{ref}}) := U^{*j}(x^j_0(t), v^j(t), U^j_{\text{ref}}, x^j_{\text{ref}}(t), y^j_{\text{ref}}(t), u^j_{\text{max}}(t)),
\]

and the value function

\[
    V^j(U^j_{\text{ref}}) := J^{*j}(x^j_0(t), v^j(t), U^j_{\text{ref}}, x^j_{\text{ref}}(t), y^j_{\text{ref}}(t), u^j_{\text{max}}(t)),
\]

as functions of only control signal reference of \( j \)-th turbine, \( U^j_{\text{ref}} \). Note that wind turbines
operate in different operating conditions, and therefore have different models and constraints.
Consequently, in general, for \( i \neq j \) we have \( z^i(\cdot) \neq z^j(\cdot), V^i(\cdot) \neq V^j(\cdot) \).

The global control objective for all turbines, at any time instant \( t \), is to produce the
power demanded by the Transmission System Operator

\[
    \sum_{j=1}^{N_{\text{ref}}} P^j(U^j_{\text{ref}}) = P^{TSO}_{\text{w}}(t), \ \forall t,
\]

where

\( N_{\text{w}}^{\text{ref}} \) denotes the number of working wind turbines in a wind farm,
\( P^j \) denotes the power of the \( j \)-th wind turbine (the power of the wind turbine is considered a part of the wind turbine’s output vector \( y^j \)), and

\( P^\text{TSO} \) is the power demand provided by the TSO.

The reconfiguration algorithm should, by redistributing the references \( U^j_{\text{ref}} \), find the smallest total cost for which the condition (3.8) is satisfied, i.e., one has to solve the following problem

\[
\begin{align*}
\min_{U^1_{\text{ref}}, \ldots, U^{N^\text{on}}_{\text{ref}}} & \quad \sum_{j=1}^{N^\text{on}} V^j(U^j_{\text{ref}}) \\
\text{subj. to} & \quad \left\{ \sum_{j=1}^{N^\text{on}} P^j(U^j_{\text{ref}}) = P^\text{TSO} \right. \\
& \quad \left. x^j(t) \quad v^j(t) \quad U^j_{\text{ref}} \quad x^j_{\text{ref}}(t) \quad y^j_{\text{ref}}(t) \quad u^j_{\text{max}}(t) \right\} \in \Theta^j, \quad j = 1, \ldots, N^\text{on}
\end{align*}
\]

(3.9)

After solving (3.9) we can also compute the optimizers \( z^j \) from (3.6) and apply them to the wind farm.

The expression (3.9) is a convex optimization problem with strictly convex, piecewise quadratic cost over polyhedral constraints. Such problems can be, in principle, solved with some type of a gradient or Newton based approach. At the points where gradient is not a continuous function one may have to compute subgradients, cf. [10]. Note that (3.9) can be readily formulated as an MIQP and then solved either by enumeration (as it is done for now) or some existing (branch & bound) MIQP solvers [34].

### 3.3. Reference generation

The supervisory controller trades-off between production and loading in the wind farm in an optimal way. It operates with a longer sampling time than the reconfigurable controller. Since the reconfigurable controller is not in charge of the overall wind farm optimization it should track the references provided by the supervisory controller.

Note that it is not clear which values of the state and outputs should be used as references between two sampling instants of the supervisory controller. Generally, the references will be obtained by some kind of interpolation between the first two prediction instants of the supervisory control:

\[
W^T_{\text{ref}}^j = \left[ U_{\text{ref}}^j \quad X_{\text{ref}}^j \quad Y_{\text{ref}}^j \right]' = f(W^T_{\text{SC}}^j, t \mod T^\text{SC}),
\]

(3.10)

where \( t \) denotes time and time instant 0 denotes the time of last supervisory control computation. The vector \( W^T_{\text{SC}}^j \) is provided by supervisory controller:

\[
W^T_{\text{SC}}^j = \begin{bmatrix}
U^j_{\text{SC}}(0) \\
X^j_{\text{SC}}(0) \\
Y^j_{\text{SC}}(0) \\
U^j_{\text{SC}}(T^\text{SC}) \\
X^j_{\text{SC}}(T^\text{SC}) \\
Y^j_{\text{SC}}(T^\text{SC})
\end{bmatrix},
\]

(3.11)
where $U^j_{\text{SC}}, X^j_{\text{SC}}$ and $Y^j_{\text{SC}}$ are input, state and output prediction vectors for j-th wind turbine used by the supervisory controller.

We note here that if the supervisory control algorithm is designed in a way that it can only provide the reference for the control signal (and not for the state and output), or a model used by the supervisory controller contains different state variables or lacks precision due to its simplicity, the algorithm described in Part I. Alg. 4.1 will be used to determine the state and output references based on the control signal provided by supervisory controller and the measurements and predictions of the disturbance used by supervisory control for last computation.

### 3.3.1. Softening the constraints

The problem (3.9) may be infeasible. Namely, there are situations when the power demanded by the TSO cannot be delivered by the wind farm. Therefore, it makes sense to consider the following relaxation

$$
\min_{U^1_{\text{ref}},\ldots,U^N_{\text{ref}}} \sum_{j=1}^{N_{\text{on}}} V^j(U^j_{\text{ref}}) + \rho \cdot \varepsilon^2
$$

subj. to

$$
\begin{aligned}
\varepsilon &\geq \sum_{j=1}^{N_{\text{on}}} P^j(U^j_{\text{ref}}) - P^T_{\text{TSO}} \\
\varepsilon &\geq -\sum_{j=1}^{N_{\text{on}}} P^j(U^j_{\text{ref}}) + P^T_{\text{TSO}} \\
x^j(t), v^j(t), U^j_{\text{ref}}, x^j_{\text{ref}}(t), y^j_{\text{ref}}(t) &\in \Theta^j, \quad j = 1, \ldots, N_{\text{on}}
\end{aligned}
$$

(3.12)

where $\varepsilon \in \mathbb{R}$ is a relaxation variable, and $\rho \in [0, \infty)$ is an adjustable, relative, weighting coefficient.

Note that the problem (3.12) is (again) a convex optimization problem with strictly convex, piecewise quadratic cost over polyhedral constraints. Therefore, we may use a modified gradient or Newton based approaches [10], or formulate it as an MIQP and solve it by enumeration or with MIQP solvers [34].

### 3.4. Simulation results

The described reconfigurable control approach was tested by numerous simulations. Some of the results are presented in this section. Three characteristic operation scenarios are presented:

1. sudden shut down of one of wind turbines during steady wind;
2. large wind gust;
3. wind farm operation during turbulent wind.

The NREL model developed in Part I of this report is used in the simulations. Since the supervisory controller is not developed yet and we do not know what kind of the model
it will use, the reference generation part of the algorithm is put aside for the moment. In the simulations we assume that the reference obtained from the supervisory control is constant for the entire simulation time.

The simulation results are shown on the figures below. The figures show the comparison between the behavior of each of the turbines with and without reconfigurable control extension (i.e. when wind turbines behave only according to the supervisory control demands). The responses with reconfigurable control extension are shown with solid line while the responses of only supervisory control are shown with dashed line. The references provided by the supervisory controller are denoted with a dotted line.

The first scenario examines a sudden shut down of one of the turbines (WT2) in a time instant between two sampling instants of the supervisory controller.

As it can be seen from Fig. 3.4 - 3.6 in the case of a wind turbine sudden shut down reconfigurable control extension enables wind farm to keep tracking the overall wind farm power reference. This is achieved by increase of the power reference of the operating turbines. The redistribution of wind farm power among operating turbines is done considering the local wind turbine operating conditions and constraints. Without reconfigurable control extension shut down of a wind turbine causes a consequent drop in the wind farm production since supervisory controller is not aware of the missing turbine until the next sampling instant.
Figure 3.5: WT shut-down – WT pitch angle, WT rotor speed

Figure 3.6: WT shut-down – WT power, WT thrust force, WT shaft moment
Another operation scenario when reconfigurable control improves wind farm operation is the sudden and large wind gust affecting one or some of the turbines. Wind farm behavior with a 4 m/s wind gust affecting only WT2 is shown on Figs. 3.7 – 3.9.

Unlike the previous scenario, in the event of large wind gust wind farm overall power is maintained even without reconfigurable control extension. This is possible because local wind turbine controller reacts upon wind gust and regulates wind turbine speed and power back to their reference values (demanded by the wind farm supervisory controller). The introduction of the reconfigurable control extension improves the transient behavior of wind turbines in the farm. This improvement is manifested through less oscillatory transients of wind turbine variables such as thrust force, rotor speed, pitch angle and shaft moment. As it can be seen on Fig. 3.9, the maximum and steady state values of the shaft moment and thrust force on some of the turbines are increased with introduction of reconfigurable control extension what might look like worsening of the wind farm behavior. However, it should be remembered that reconfigurable control extension was designed with the goal of fatigue reduction on wind turbines. Wind turbine fatigue is a consequence of oscillatory stress and strain resulting from oscillatory loads i.e. forces and moments. Therefore, wind turbine fatigue is not affected by the loads absolute values, but by their amplitude of change and number of periods before the transient oscillations die out. Both of these quantities are reduced with introduction of the reconfigurable control as a consequence of the chosen cost function used for the reconfigurable controller design. The demonstrated behavior shows that in the case of large wind gusts affecting only some of the turbine wind farm will maintain its power reference even if it is controlled only with supervisory controller. However, the introduction of the reconfigurable control extension will assure minimization of the wind turbine fatigue during accommodation of the wind gusts. The increase of the absolute values of some of the loads poses no problem for wind turbine structure since it has to be designed to withstand steady loads several times larger than nominal loads (see e.g. [35]). The reduction in the loads amplitude and more aperiodic behavior reduces the wind turbine fatigue what is beneficial.

The third test scenario covers the wind farm operation during (highly) turbulent wind. The behavior of all wind turbines in the farm is shown in Figs. 3.10 - 3.12.

In this scenario there is neither wind turbine fault nor extreme wind gust. However, wind temporal and spatial variations make it considerably different than the wind predicted by the supervisory controller. Due to presence of local wind turbine controllers, wind farm is again able to deliver demanded power. This requires significant effort from the local controller, that manifests in very active blade pitching. The reconfigurable control extension can improves this situation because it is aware of the wind speed changes at higher frequency than the supervisory controller. Therefore it can adjust the power reference for particular turbines dynamically between two sampling instants of the supervisory controller. The improvement is visible form the reduction in standard deviation of pitch angle and the loads (thrust force and shaft moment). This results in fatigue reduction what was one of the objectives for reconfigurable controller design.
Figure 3.7: Wind gust – WT wind speed, WT operational state, WF production

Figure 3.8: Wind gust – WT pitch angle, WT rotor speed
Figure 3.9: Wind gust – WT power, WT thrust force, WT shaft moment
Figure 3.10: Turbulent wind – WT wind speed, WT operational state, WF production

Figure 3.11: Turbulent wind – WT pitch angle, WT rotor speed
Figure 3.12: Turbulent wind – WT power, WT thrust force, WT shaft moment
Chapter 4

Conclusion

In Part II of the Deliverable the reconfigurable control problem was defined and its solution was provided. The reconfigurable controller has a task to react when sudden disturbances occur, e.g. a wind gust or wind turbine shut-down. In such scenarios the response of the wind farm controlled by the supervisory controller is compromised due to slow sampling time of the supervisory controller. Since the supervisory controller for a large wind farm is supposed to solve a very complex optimization problem with long prediction horizon, the sampling time of the supervisory control can not be reduced.

The proposed solution to the reconfigurable control problem is based on multi-parametric solution of the Constrained Finite Time Optimal Control problem for each wind turbine – the optimal control action and its associated cost are obtained as explicit expressions in parameters that present measurements, references and non-constant system constraints. Thus, most of computational effort is moved off-line. By using the multi-parametric solutions to local control problems and disregarding the coupling between the wind turbines introduced by the common wind field, the remaining on-line computation becomes much simpler than the overall wind farm optimization. Therefore, it can be run at faster time scale. The simulations of the presented control solutions for different scenarios show promising results.

In the presented work the most straightforward formulation of CFTOC and reconfiguration algorithm was used. In further work different formulations will be analyzed and evaluated according to the on-line computational effort that their solving requires. The aim is to find a formulation that meets the control objective and requires as low as possible on-line computational effort. Also, the supervisory controller and the reconfigurable controller will be merged, which will require a conceptual solution for generating references for reconfigurable control.
Appendix A

Notation

Part I

\( v \) wind speed, [m/s]
\( P_{\text{dem}} \) the power demand provided by the user (wind farm control input), [kW]
\( P \) produced electrical power, [kW]
\( \beta \) pitch angle, [\(^\circ\)]
\( \omega_r \) rotational velocity of wind turbine rotor, [rad/s]
\( \omega_g \) rotational velocity of generator rotor, [rad/s]
\( T_r \) turbine rotor torque, [Nm]
\( T_g \) electrical generator torque, [Nm]
\( \beta_{\text{ref}} \) reference for the pitch angle, [\(^\circ\)]
\( \omega_{\text{ref}} \) reference for the generator speed, [rpm]
\( T_{\text{ref}} \) reference for the generator torque, [kW]
\( P_{\text{ref}} \) power reference provided to generator power controller, [kW]
\( \beta_{\text{meas}} \) measured pitch angle, [\(^\circ\)]
\( \omega_{\text{meas}} \) measured generator speed, [rpm]
\( v_{\text{meas}} \) measured wind speed, [m/s]
\( \omega_{\text{min}} \) minimal generator speed necessary for production of electrical power, [rpm]
\( \omega_{\text{nom}} \) nominal generator speed, [rpm]
\( \beta_{\text{opt}} \) optimal pitch angle – the pitch angle at which maximal power is produced at given tip-speed ratio, [\(^\circ\)]
\( \lambda_{\text{opt}} \) optimal tip-speed ratio – the tip-speed ratio at which maximal power is produced at given pitch angle, [-]
\( T_{\text{opt}} \) optimal generator torque – determined by the controller (NREL), [Nm]
\( K_P, K_I \) parameters of generator speed controller that uses pitch angle as control variable,
\( P_a \) wind turbine aerodynamic power, [kW]
\( P_a^{\text{max}} \) maximal available aerodynamic power, [kW]
\( P_{\text{max}} \) maximal electrical power that can be produced at given time, [kW]
\( F_t \) thrust force, [N]
\[ M_{\text{shaft}} \] shaft moment, [Nm]
\[ C_P \] power coefficient, [-]
\[ C_T \] thrust coefficient, [-]
\[ C_Q \] torque coefficient, [-]
\[ v_{\text{eff}} \] effective wind speed experienced by the rotor, [m/s]
\[ \lambda \] tip speed ratio, [-]
\[ a \] axial induction factor, [-]
\[ x \] position of the nacelle in mean wind direction, [m]
\[ M_t \] modal mass of the tower, [kg]
\[ D_t \] modal damping of the tower, [Ns/m]
\[ C_t \] modal stiffness of the tower, [N/m]
\[ R \] wind turbine rotor radius, [m]
\[ H \] nacelle height, [m]
\[ B \] main shaft viscous friction, [Nms/rad]
\[ K \] main shaft spring constant, [N/rad]
\[ \rho \] air density, [kg/m³]
\[ n_{gb} \] gear ratio, [-]
\[ J_r \] rotor inertia, [kg/m²]
\[ J_g \] generator inertia, [kg/m²]
\[ J_e \] equivalent win turbine inertia, [kg/m²]
\[ C_{Tg} \] coefficient that describes torque dependant drive train losses (V80), [-]
\[ C_{\omega g} \] coefficient that describes speed dependant drive train losses (V80), [rpm]
\[ T_{\text{gen}} \] time constant of the V80 generator model, [s]
\[ \text{FULL} \] logic signal used in V80 controller, [-]
\[ K_P^g, K_I^g \] parameters of V80 power controller, [-]
\[ \mu \] efficiency of the NREL generator, [-]
\[ \mu_{\text{ctrl}} \] compensation for efficiency of the NREL generator used in the controller, [-]
\[ p \] number of poles in electrical generator, [-]
\[ s \] electrical generator slip, [-]
\[ \omega_S \] frequency of stator power supply, [rad/s]
\[ P_{\text{stat}} \] stator power, [kW]
\[ P_{\text{rot}} \] rotor power (of the electrical generator), [kW]
\[ K_{PT} \] coefficient that describes proportionality between stator power and generator torque, [-]

\( a_1, b_1, a_2, b_2, K_{\text{opt}}, \omega_{g,\text{opt}}, \omega_{g,\text{Opt}} \) parameters of the NREL optimal torque characteristic, see (4.22)
\( v_{\omega_{\text{min}}}, v_{\omega_{\text{max}}}, k, l, T_{\text{sc}} \) parameters of the V80 stationary control, see (3.1.1)
\( K^D, T_1^D, T_2^D \) parameters of the V80 UMPDAMP subsystems, see (2.24)
Part II

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>θ</td>
<td>parameter vector of the problem, $\theta = [x_0' \ v' \ u_{ref}' \ x_{ref}' \ y_{ref}' \ u_{max}']$</td>
</tr>
<tr>
<td>$x_0$</td>
<td>system state</td>
</tr>
<tr>
<td>$v$</td>
<td>wind speed</td>
</tr>
<tr>
<td>$u_{ref}$</td>
<td>input reference</td>
</tr>
<tr>
<td>$x_{ref}$</td>
<td>state reference</td>
</tr>
<tr>
<td>$y_{ref}$</td>
<td>output reference</td>
</tr>
<tr>
<td>$u_{max}$</td>
<td>upper bound on system input (maximal available power)</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>set of feasible parameters</td>
</tr>
<tr>
<td>$J^*$</td>
<td>value function, $J^* : \Theta \to \mathbb{R}$</td>
</tr>
<tr>
<td>$z^*$</td>
<td>the optimizer function, $z^* : \Theta \to \mathbb{R}^{n_z}$</td>
</tr>
<tr>
<td>$j$</td>
<td>index that denotes the number of wind turbine</td>
</tr>
<tr>
<td>$P_j$</td>
<td>power produced by the $j$-th wind turbine, [kW]</td>
</tr>
<tr>
<td>$N_{on}$</td>
<td>number of the operational wind turbines in the wind farm,</td>
</tr>
<tr>
<td>$P_{WF}^{TSO}$</td>
<td>wind farm power demand provided by TSO, [kW]</td>
</tr>
<tr>
<td>$WT_{ref}^{SC}$</td>
<td>wind farm references provided by supervisory controller, [kW]</td>
</tr>
<tr>
<td>$WT_{ref}$</td>
<td>wind farm references created by reconfigurable control based on supervisory controller references, [kW]</td>
</tr>
</tbody>
</table>
Bibliography


